Platform Entry and Vendor Competition in On-Demand Economy

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We consider an on-demand delivery system with multiple competing vendors and examine the impact of introducing a delivery platform on their competition. Customers have preferences for differentiated vendors while also being sensitive to the overall cost of both the food price and the delivery fee. They place orders from a vendor only if they can be delivered within a specified delivery window. Each vendor strategically decides whether to participate on the platform, build its dedicated delivery fleet (i.e., employing in-house delivery), or not offer any delivery option (i.e., exclusively serving local customers as an outside option) to maximize its profit. Vendors who build their own dedicated delivery fleets decide on both the food price and delivery fee. Vendors who choose to participate on the platform set their food price, while the profit-maximizing platform sets the delivery fee and takes a commission from the vendors' revenue. We solve for the system equilibrium and benchmark it against the system without the platform. Our findings indicate that the introduction of the platform either intensifies vendor competition, leading to lower vendor profits and higher individual customer surplus, or conversely, alleviates vendor competition, leading to higher vendor profits and lower individual customer surplus. The intensifying or alleviating role of the platform on vendor competition hinges on the competitive environment of the market in which the vendors operate. We characterize conditions under which the platform's equilibrium strategy involves subsidizing per-order delivery to encourage vendor participation. However, perhaps surprisingly, the seemingly appealing per-order delivery subsidy (offered by the platform to attract vendor participation) hurts vendors by encouraging them to engage in more intensified competition, ultimately diminishing their profits. Conversely, if the platform derives profits from per-order delivery, introducing the platform alleviates vendor competition and benefits vendors. Therefore, the platform-enabled sharing of couriers can improve delivery efficiency and lower delivery costs but may intensify vendor competition through per-order delivery subsidy, placing them in a prisoner's dilemma that ultimately lowers their profits. These insights also apply to an e-commerce platform that provides storage and fulfillment services with economies of scale enabled by aggregating the business of sellers operating on the platform.

1. Introduction

Having become an integral part of modern life, on-demand delivery services brought a new level of convenience and changed how businesses operate. Although the on-demand delivery service is not a new concept, recent technological advancements in smartphones, e-payment, and real-time tracking, among others, have enabled the on-demand delivery service to transition from dial-up (traditionally, on-demand delivery services were accessible through phone calls directly to vendors) to online and led to the emergence of third-party platforms, like Doordash and Uber Eats, which specialize in facilitating these delivery services. Now, with just a few taps on a smartphone, customers can have a wide variety of products, from restaurant meals and groceries to everyday essentials, delivered right to their doorsteps. In 2023, the value of the grocery delivery segment was estimated to be around 183 billion U.S. dollars, and the revenue from meal delivery services was approximately 87 billion dollars.[1]

Despite the rapid expansion of on-demand delivery platforms, the relationship between vendors and third-party delivery platforms is often fraught with tension. Vendors face a dilemma: despite the substantial share of their earnings taken by the platforms, they feel compelled to use them to stay competitive. "I used to do just fine with my own delivery fleet, but now I feel that opting out of Seamless[2] is not an option" said by Palombino, a local restaurant operator (Bronner 2020). This paradoxical situation highlights the intricate interplay within the vendor-platform relationship, in which the need to remain market-relevant through platforms is juxtaposed against the cost of selling through a platform.

In this work, we use food delivery as a motivating example (though our model is broadly applicable to other areas, such as grocery delivery and e-commerce fulfillment) to construct a stylized model that examines the impact of introducing a delivery platform on vendor competition. Specifically, we investigate an on-demand delivery system with multiple competing vendors. Customers have preferences for differentiated vendors (e.g., based on their cuisine type) while also being sensitive to the overall cost of both the food price and the delivery fee. In addition, customers place orders from a vendor only if their deliveries can be made within a specified delivery window. Each vendor strategically decides whether to participate on the platform, build its dedicated delivery fleet (i.e., employ in-house delivery), or not offer any delivery option (i.e., exclusively serving local customers as an outside option) to maximize its profit. Vendors who build their own dedicated delivery fleets decide on both their food price and delivery fee. Vendors who choose to participate

[1] https://www.statista.com/forecasts/891070/eservices-revenue-for-selected-countries-by-segment

[2] Seamless is an online food ordering service launched in 1999 and merged with Grubhub in 2013.

on the platform set their food price, while the profit-maximizing platform sets the delivery fee and takes a commission from the vendors' revenue. Service providers (either the platform or vendors establishing dedicated delivery fleets) procure service capacity at a fixed cost per unit per unit of time, which we refer to as the marginal delivery cost. We assume that the delivery time associated with a service provider benefits from economies of scale. By aggregating demand and supply, the platform can enhance delivery efficiency by operating on a larger scale, thereby increasing courier utilization and reducing the average delivery cost per unit of demand required for meeting the delivery window constraint.

In our base model, we consider a setting in which the platform tends to have stronger market power than vendors. We capture this by letting the platform and vendors move sequentially. Specifically, the sequence of events is as follows: In stage 1, the platform commits to a delivery fee to charge customers. In stage 2, vendors simultaneously determine whether to participate on the platform and their food price (if they participate) or their full (i.e., food plus delivery) price (if they do not participate). We benchmark this setting against one in which there is no platform, and each vendor runs its own delivery service, setting both their food price and delivery fee to maximize profits (or does not offer any delivery services). Vendors may have access to two customer channels: walk-in and delivery customers. In our analysis, we assume that these two channels are independent and focus exclusively on the operations of the delivery channel (in practice, vendors typically manage these channels separately and set different prices for each, see, e.g., Rana and Haddon 2023).

We uncover the nuanced impacts of introducing the platform on vendor competition. Depending on the market's competitive environment, the platform either opts to subsidize the delivery service per order by setting a delivery fee lower than the marginal delivery cost or it opts to profit per order from the delivery service by setting a delivery fee higher than the marginal delivery cost. Perhaps paradoxically, the introduction of the platform hurts vendors if the platform chooses to subsidize the delivery service per order, while it benefits vendors if the platform chooses to profit per order from the delivery service, provided that vendors opt to operate in the delivery channel by building their in-house delivery fleets in the absence of the platform. (Otherwise, the introduction of the platform may benefit vendors by reducing barriers for vendors who otherwise do not offer delivery service.)

We explain the above results as follows. When deciding whether to participate on the platform, vendors weigh the profit of building their own dedicated delivery fleets against that of participating on the platform. Specifically, vendors building dedicated delivery fleets must procure sufficient service capacity to meet the delivery window, whereas vendors participating on the platform avoid the need to procure service capacity but must share revenues with the platform. Therefore, we differentiate between the following two scenarios, which lead to contrasting strategies of the platform in equilibrium and its subsequent implications of the platform entry.

Under conditions in which the marginal delivery cost is low, the delivery window is long, the market potential for each vendor is large, or the level of substitutability among vendors is low, the platform chooses to subsidize the delivery service per order. This decision stems from the fact that, under these conditions, vendors possess compelling incentives to establish their own dedicated delivery fleets. This incentive is driven by the relative affordability of maintaining the service standard, the sizable market potential for each vendor, or the less intense competitive landscape. Therefore, to attract vendors, the platform needs to subsidize the delivery service per order by setting the delivery fee below the marginal delivery cost, making it more appealing to vendors. In this case, all vendors choose to participate on the platform in equilibrium. However, when the platform subsidizes the delivery service per order, its introduction lowers the operational costs for vendors participating on the platform and gives them a competitive advantage. This encourages vendors to reduce the overall prices to attract more customers. Consequently, the introduction of the platform intensifies the competition among vendors, resulting in lower vendor profits and higher individual customer surplus.

In contrast, when facing conditions with a high marginal delivery cost, a short delivery window, limited market potential for each vendor, or high levels of substitutability, the platform derives profit per order from delivery service by charging a delivery fee higher than the marginal delivery cost. This strategy is adopted because, under such conditions, participating on the platform is more favorable for vendors. This preference is driven by factors including avoiding the substantial costs of maintaining the service standard, operating with a market potential that does not justify the investment in dedicated delivery fleets, or sidestepping the intense competition. Therefore, it allows the platform to capitalize on economies of scale resulting from aggregating demand and supply, enabling it to charge a higher delivery fee while still ensuring the participation of certain vendors. Then, the platform's priority shifts from attracting vendors to revenue generation through its delivery service, which might result in asymmetric equilibrium outcomes with some vendors participating on the platform while others build dedicated delivery fleets. When the platform profits per order from delivery service, it raises the operational costs for vendors and reduces the competitive advantage for those vendors who participate on the platform. These vendors find it beneficial to refrain from aggressive competition and increase the food prices (and thus the overall prices) paid by customers. Similarly, vendors who choose not to participate on the platform also increase their prices to capitalize on the increased demand that the platform leaves behind. In this case, the introduction of the platform alleviates vendor competition, resulting in higher vendor profits and lower individual customer surplus.

These findings reveal that the decision of a platform to either subsidize or profit from the delivery service per order is contingent upon the competitive business environments, which determine the vendors' preference to participate on the platform or build dedicated delivery fleets. This strategic decision by the platform is closely linked to the competition among vendors and, consequently, the welfare of vendors and customers. The seemingly appealing per-order delivery subsidy (offered by the platform to attract vendor participation) hurts vendors by compelling them to engage in more intensive competition, ultimately diminishing their profits. Therefore, the sharing of couriers enabled by the platform can improve delivery efficiency and lower delivery costs, but it may intensify vendor competition through per-order delivery subsidy and place them in a prisoner's dilemma that ultimately lowers their profits.

We also explore various extensions. First, we consider alternative contracts between the platform and competing vendors. The platform can choose to offer a fixed participation fee to reward vendors for their participation or impose a per-order transaction fee on vendors. We discuss the role played by the agreement between the platform and vendors in determining the impact of platform entry. Our result indicates that the core impact of the platform's introduction continues to revolve around its strategy on the delivery and per-order transaction fees, which, in turn, either intensifies (with low delivery and transaction fees) or alleviates (with high delivery and transaction fees) the competition among vendors. We show that it is possible to design contracts that steer the platform's decisions in a desired way. For example, under a transaction-based contract in which the platform charges vendors only a per-order transaction fee without sharing vendors' revenues, the introduction of the platform always benefits vendors (while hurting customers). These insights can be helpful to regulators overseeing the delivery service industry and entities operating delivery platforms.

Second, we examine the significance of relative market power in shaping the impact of the platform entry. The relative market power between the platform and vendors is captured through the decision-making process. In our base model, we assume the platform possesses greater market power than the vendors. This is reflected in a sequential decision-making process: the platform commits to a delivery fee in the first stage, and then vendors simultaneously determine whether to participate on the platform and the associated pricing strategies in the second stage. In this setting, we demonstrate that the introduction of the platform can either intensify vendor competition, harming vendors while benefiting customers or, conversely, alleviate vendor competition, benefiting vendors at the expense of customers. In a scenario in which the platform and vendors tend to have equal market power, which is modeled by simultaneous moves of the platform and vendors, the introduction of the platform can no longer intensify the competition among vendors. In this case, relative to the system without the platform, vendor profits are higher, and the customer surplus is lower when an equilibrium exists.

Third, we expand our analysis by considering the system with asymmetric vendor scales, for which our main findings are robust. Specifically, we identify sufficient conditions under which the introduction of the platform yields either subsidizing or profiting from per-order delivery service. In addition, we show that small-scale vendors are more inclined to participate on the platform and are more likely to benefit from the platform entry as compared to large-scale vendors. In contrast to our findings from the base model, it is possible for small-scale vendors to gain higher profits with platform entry, even if it intensifies vendor competition. This is because, to attract vendors of relatively larger scales, the platform must set a delivery fee lower than those aimed at recruiting smaller-scale vendors. Consequently, the reduction in the delivery fee may benefit smaller-scale vendors and improve their profits to a greater extent.

Lastly, we investigate an alternative setting where couriers receive a piecemeal take-home pay per order but do not receive compensation for idle time while waiting for dispatch. In this setting, we demonstrate that the introduction of the platform always intensifies vendor competition, leading to lower vendor profits and higher individual customer surplus. By not compensating couriers during idle periods, vendors can eliminate the costs associated with maintaining delivery capacity. This reduces the barrier for vendors to establish their own dedicated delivery fleets. Consequently, the platform must subsidize the per-order delivery service to enhance its attractiveness to vendors, thereby intensifying vendor competition.

2. Literature Review

Broadly speaking, our work adds to the emerging body of literature on on-demand service platforms. See Benjaafar and Hu (2020), Chen et al. (2020), and Hu (2021) for general reviews and the references therein. More specifically, our work is closely related to the stream on on-demand delivery services. Below we review papers of this particular stream.

First, there is a stream of literature that explores the operational problems faced by on-demand delivery platforms. Chen and Hu (2023) investigate and contrast two order dispatching rules: one that delivers a single order per trip and the other that delivers a consecutive batch of orders.

They characterize the conditions under which one dispatching rule is more effective than the other. Gorbushin et al. (2023) investigate a system consisting of two vendor hubs situated at opposite ends of a Hotelling line, considering customers' heterogeneous preferences and locations along the line. The authors explore the "in-region policy," whereby drivers return to their original hub after a delivery, versus the "cross-region policy" which directs drivers to the nearer hub after a delivery. Cao et al. (2024) consider a system in which a platform operates with multiple kitchens. Customers can place orders for dishes from various kitchens within a single delivery. They characterize the performance of such a system, specifically the delays and the costs of routing for collecting dishes. Both Zhao et al. (2024) and Liu et al. (2020) investigate the order-driver matching policies within a food delivery system. Zhao et al. (2024) focus on incorporating the uncertain food preparation times as a critical component of their analysis, while Liu et al. (2020) propose a framework that integrates unobservable driver routing behavior and highly uncertain service times with optimization tools for efficient order-driver assignments.

Bo et al. (2024) and Liu et al. (2023) examine the optimal staffing (and pricing) in on-demand delivery systems. They both highlight the spatial feature inherent in such a system. Bo et al. (2024) obtain a lower bound for the average cost per unit of time and show that this lower bound is only achievable with the power of 2/3 safety level staffing rule (i.e., the safety staffing level is proportionate to the power of 2/3 of the nominal workload). They then propose a dispatching policy that asymptotically achieves this lower bound. Liu et al. (2023) utilize a spatial queuing model to investigate a delivery system in which the decisions on whether to participate on the platform by all stakeholders—customers, couriers, and restaurants—are endogenously determined, and solve for the equilibrium in the Heavy-Traffic Regime. Notably, we also endogenize vendors' decisions regarding whether to participate on the platform, similar to Liu et al. (2023). However, we do so in the context of vendor competition, while Liu et al. (2023) do not consider the competition effect and assume each vendor has its exclusive market base. In contrast to this stream of literature, the focus of this paper is on the impact of introducing a delivery platform on vendor competition. We emphasize how platform entry alters vendors' operational choices and pricing strategies and how these changes affect vendor profits and customer surplus rather than examining the detailed operational-level policies of the platform.

Second, there is a stream of literature that studies various contract forms between food delivery platforms and restaurants. Both Feldman et al. (2023) and Chen et al. (2022) consider a setting in which a restaurant serves customers via two channels: dine-in customers and food delivery customers, and the delivery service is facilitated by a third-party platform. They design contracts that "coordinate" the system, maximizing the overall profit of the two channels. Chen et al. (2022) also highlight the potential operational inefficiency caused by the high accessibility of delivery services and the large size of the delivery worker pool. Oh et al. (2023) examine a system with restaurants at various locations selling to customers at a single location through a platform. Restaurants can bid for a featured spot on the platform by choosing a higher commission rate. They design contracts to nudge restaurants to bid "correctly" based on their distances to customers, ensuring the system is coordinated. In this work, to emphasize our focus on the impact of introducing a delivery platform on vendor competition, we examine the role played by contracts. In contrast to the earlier literature in this stream, we study contracts between the platform and vendors within a competitive setting.

Third, there is a body of empirical research that delves into various aspects of the food delivery industry. Mao et al. (2022) provide guidelines on the operations of on-demand meal delivery platforms and describe the empirical research opportunities. Li and Wang (2024a) evaluate the effectiveness of regulating the commission fees charged by the platform. Li and Wang (2024b) investigate how the introduction of the delivery channel affects takeout and dine-in channels, as well as its overall impact on restaurants. Zhang et al. (2023) first derive a theoretical model that allows them to develop hypotheses on the impacts of restaurant density on sales volume, revenue, and average spending per order and then empirically test each hypothesis. In Zhang et al. (2023)'s analytical model, they consider the operational performance of a single restaurant, while our model considers the interplay between the platform and multiple competing vendors. In contrast to this stream of literature, we derive analytical results on the impact of introducing a delivery platform on vendor competition.

Fourth, our work is related to the literature on two-sided competition in the context of the sharing economy. When vendors operate with their own dedicated delivery fleets, the competition among vendors/the platform resembles the competition between firms for customers and service providers in a two-sided marketplace. Cohen and Zhang (2022) examine the impact of "coopetition" between two firms—in which competing platforms introduce a joint service and share profits—on customers, service providers, and the platforms themselves. Bai and Tang (2022), Bernstein et al. (2020), and Benjaafar et al. (2020) investigate two-sided competition between two firms, where customer utility is determined by price and delay, and service provider utility is determined by wage and utilization. Bai and Tang (2022) investigate the conditions under which two competing firms can be profitable in equilibrium. Bernstein et al. (2020) consider competition between two firms in two scenarios:

one in which service providers are single-homing and one in which service providers are multi-homing. They compare the equilibrium outcomes in these scenarios and find that multi-homing may lead to worse outcomes for all stakeholders. Benjaafar et al. (2020) characterize conditions under which competition between two firms can result in worse outcomes for both customers and service providers compared to a monopoly. Both Bernstein et al. (2020) and Benjaafar et al. (2020) demonstrate that allowing providers to multi-homing alleviates firm competition and may lead to worse outcomes. Nikzad (2022) shows that the size of the labor pool affects the impact of competition on customers and providers. When the labor market is thin, firms compete fiercely for providers, which can lead to higher prices for customers. Unlike these studies, in this paper, we consider the possibility of outsourcing tasks to a third-party platform instead of recruiting their own service providers and endogenize such decisions made by vendors to examine the impact of introducing such an option on their competition.

Lastly, this work is related to the stream of literature that studies platform market entry (Belleflamme and Martin 2021). Notably, the system examined in our paper shares many features with the "Fulfillment by Amazon (FBA)" program launched by Amazon. This program allows third-party sellers to access Amazon's full set of supply chain services by paying a referral fee (typically between $8-15\%$ of revenue), a selling fee per unit of sold product, and a storage fee per unit of unsold product.[3] Between our system and the FBA system, the interactions between the platform (or Amazon) and vendors (or third-party sellers) are analogous, and the platform (or Amazon) may take advantage of economies of scale enabled by aggregating the business of vendors (or sellers).

However, these two systems also have distinctions. Perhaps the most notable difference is that in the system we consider, the platform solely provides delivery services to vendors and does not directly compete in the same business, whereas Amazon competes with third-party sellers in the FBA program. The work most closely related to ours is Lai et al. (2022), which investigates the competition between Amazon and a single third-party seller in the FBA system. As Lai et al. (2022) do not consider the competition among third-party sellers, the seller consistently benefits from the FBA program, provided its willingness to participate. In contrast, in our setting, due to the intricate competitive dynamics among vendors, vendors may find themselves worse off with the introduction of the platform, even if they choose to participate on the platform by themselves (i.e., a prisoner's dilemma).

[3] `https://sell.amazon.com/blog/3pl-third-party-logistics`

3. Base Model

In this section, we describe our base model. We formulate the problem in Section 3.1 and describe the equilibrium characterization in Section 3.2.

3.1. Model Steup

Consider $n \geq 2$ vendors who offer catering services to customers. In what follows, we present the details of our model. Although we describe our model in the context of food delivery, it is broadly applicable to other areas, such as grocery delivery and e-commerce fulfillment.

Customers. We consider a market with differentiated food options in which customers exhibit preferences to vendors while also being sensitive to the overall cost of food price and delivery fee. Vendors are assumed to be symmetric in the base model in the eye of customers, and we will consider asymmetric vendors in Section 5.4. For convenience, we refer to the summation of the food price and the delivery fee as the *full price*, and we use p_i to denote the full price associated with vendor i, where $i \in \{1, \ldots, n\}$. We characterize the demand rate associated with vendor i as

$$\lambda_i = \left(\theta - \alpha p_i + \beta \sum_{j \neq i} p_j \right)^+ \quad \text{and} \quad (n-1)\beta < \alpha, \tag{1}$$

where $\theta > 0$ represents the market potential for each vendor (i.e., the demand rate when all vendors offer the service for free), $\alpha > 0$ reflects the price sensitivity, and $\beta > 0$ reflects the cross-price sensitivity of demand. Here, β can serve as an indicator of the substitutability among vendors. A thorough discussion on the level of substitutability is postponed to Section 5.1. Without loss of generality, we normalize $\alpha = 1$. Following the treatment in the literature (e.g., Allon and Federgruen 2007 and Federgruen and Hu 2015), we assume that $(n-1)\beta < \alpha$. This condition is usually referred to as the "dominant diagonal" condition. It suggests that a uniform increase in price by all vendors does not result in a higher demand for any specific vendor, and an increase in price by a particular vendor does not lead to a rise in the system's aggregate demand.

Vendors. Vendors are strategic. Each vendor, say vendor i, where $i \in \{1, \ldots, n\}$, determines whether to offer the delivery service and (if so) whether to build a dedicated delivery fleet or sign up to use the delivery service offered by a delivery platform. If vendor i chooses to build its dedicated delivery fleet, it determines its full price p_i to charge customers (note that there is no need to distinguish the food price and the delivery fee for those vendors who build dedicated delivery fleets). If vendor i chooses to participate on the platform, it determines its food price p_i^F, while the delivery fee is determined by the platform. Let p_d denote the delivery fee charged by the platform. Then $p_i = p_i^F + p_d$ if vendor i chooses to participate on the platform.[4] Additionally, aligning with the predominant contract adopted by leading food delivery platforms (e.g., Grubhub, Doordash, and Uber Eats), we assume that the platform enforces a commission rate $\gamma \in [0, 1)$ on revenue (i.e., the food price) per order for vendors who use its service. That is, vendor i shares γp_i^F of its revenue per delivery with the platform by participating on the platform. We assume γ to be exogenous as the platform has limited power in setting the commission rate. This value is typically influenced by market standards and determined by factors such as market competition and government regulation (Haddon and Rana 2021).[5] In Section 5.3, we consider generalized contracts between the platform and vendors over the revenue-sharing commission contract.

Delivery window and service supply. To operate in the market, service providers (either the platform or vendors who build dedicated delivery fleets) must build service capacity to ensure that the deliveries are made within a delivery window $\overline{w}$. This delivery window is generally subject to market standards and common practices, often influenced by the competitive environment of the market as a whole. For example, according to a survey in year 2023, in the U.S., about 80% customers expect their online food orders to be delivered within 40 minutes.[6] In the online grocery delivery market, "Two-day delivery was once a differentiator for online merchants, but it is now a service that many companies offer" as exemplified by Ms. Wise of Forrester Research (Alcántara 2020). Let $W(\lambda, \mu)$ denote the delivery time for a system with the customer arrival rate λ and the overall service rate μ. Given $\mu > \lambda$, we assume that the (average) delivery time function adopts the following form:

$$W(\lambda, \mu) = \begin{cases} f(\mu - \lambda) & \text{if } \lambda > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $f(\cdot)$ is strictly decreasing, i.e., the delivery time increases as the gap between the supply and demand narrows. Observe that the delivery time function exhibits the feature of economies of scale (i.e., scaling up μ and λ by the same parameter results in a decrease in the delivery time), which is a crucial aspect in service systems. Then, the introduction of the platform may enhance delivery efficiency by aggregating demand and supply due to the economies of scale, thereby reducing delivery costs per unit of demand and increasing courier utilization. Although our

[4] This aligns with the common practice, where the platform charges customers a delivery fee on top of the order cost and uses it to cover delivery expenses such as courier compensation. See https://www.cnbc.com/2024/07/27/food-delivery-fees-are-rising.html.

[5] In practice, the commission rates typically fall within the range of 15% to 30% (Ahuja et al. 2021). However, in this study, as we do not consider vendors' costs, such as food preparation expenses, the commission rate γ should be higher to align with practices.

[6] https://www.statista.com/statistics/1366702/online-food-delivery-maximum-wait-time-united-states/.

analysis can accommodate a general form of delivery time function, we let $f(\mu-\lambda)=\frac{1}{\mu-\lambda}$, which is the M/M/1 queue sojourn time. This particular form of delivery time function enables us to derive simple and interpretable results regarding the impact of introducing the delivery platform on vendor competition, facilitating our discussion.

We use $\mu(\lambda;\overline{w})$ to denote the service rate required to ensure that deliveries are made within the delivery window requirement $\overline{w}$, given the demand rate λ. Then $\mu(\lambda;\overline{w})=\lambda+f^{-1}(\overline{w})$, where $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$. Then a service provider incurs a cost of $h\cdot\mu(\lambda;\overline{w})$ to build the capacity, where h can be interpreted as the marginal delivery cost following Allon and Federgruen (2007). Observe that the cost of maintaining the service standard $h\cdot\mu(\lambda;\overline{w})=h\lambda+hf^{-1}(\overline{w})$ can be decomposed into the operational cost $h\lambda$ and the fixed cost $hf^{-1}(\overline{w})$. The operational cost is determined by multiplying the marginal delivery cost by the realized demand of the service provider (i.e., either the platform or a vendor who builds its dedicated delivery fleet). The fixed cost can be interpreted as the expense associated with the buffer capacity required. The delivery cost per unit of demand, $h\cdot\mu(\lambda;\overline{w})/\lambda=h+hf^{-1}(\overline{w})/\lambda$, decreases as λ increases, i.e., the fixed cost is distributed across a larger number of customers, thereby demonstrating the efficiency gained from scale economies.

In practice, h can represent the hourly wage of a recruited courier, which depends on the per-delivery salary and the average number of deliveries completed per hour. These factors can vary based on delivery range and modes of transportation, which differ by city. For instance, couriers in Los Angeles predominantly use cars, while those in New York or Toronto typically take bikes. Table 1 provides a summary of salary data sourced from `indeed.com` and delivery range information extracted from the Uber Eats application for downtown deliveries, illustrating the variability in average salaries and delivery distances across various North American cities. Additionally, the hourly wage is related to the minimum wage rate established by the regulator and may vary depending on the region. Service providers are expected to meet these minimum wage requirements.

City	Delivery Range (miles)	Hourly Salary ($)	Transport
New York	1.2	17.42	Bike
Chicago	2.8	23.06	Bike
Los Angeles	4	20.14	Car
Toronto	1.6	17.24	Bike
Houston	3.2	18.75	Car
Phoenix	5.5	18.47	Car
Philadelphia	0.9	24.49	Bike

Table 1 Delivery Range and Hourly Salary by City

In this work, we assume that couriers receive hourly pay from either the platform or vendors building dedicated delivery fleets. For example, GoPuff has pledged to provide its couriers with a minimum hourly rate, even if they are not assigned a sufficient number of orders to deliver through the platform.[7] Moreover, regulations, such as California Assembly Bill No. 5 (AB5), pursued to reclassify gig workers as employees with employee benefits (Lazo 2019). Some platforms are implementing such schemes. For instance, depending on their employment status, Instacart compensates its drivers either on an hourly basis or per delivery.[8] In Section 5.5, we analyze scenarios where couriers receive a piecemeal take-home-pay per order but do not receive compensation for being idle and waiting for dispatch. We establish that, in this extension, the competition-intensifying effect induced by the platform entry persists.

Platform. The platform decides on the delivery fee p_d to charge customers to maximize its profits, given its presence in the market. We allow $p_d < 0$ to account for scenarios in which the platform provides customer compensation for the delivery, such as through the distribution of coupons (in this case, it is still possible for the platform to make a profit through revenue sharing, but to maintain a market presence, the platform may need to burn cash rather than to earn a profit). We formally formulate the problem faced by the platform in Section 3.2. We start by setting aside the possibility that the platform exits the market when it cannot make a profit. We use this treatment for the following reasons. First, it allows us to focus sharply on understanding how the competition among vendors is affected by the introduction of the platform. Second, this is consistent with the practice. Maintaining market presence often takes precedence over generating profit for startups. They tolerate burning cash before achieving sustainable operations. Additionally, customers' meal consumption habits can change over time. Platforms are driven to convert customers initially attracted by discounts into habitual and high-value users, even if it comes at the expense of temporarily negative profits (Haddon 2019). Indeed, major delivery platforms in the U.S., such as DoorDash and Uber Eats, have been burning cash in recent years (Ahuja et al. 2021 and Rana and Haddon 2021). Our results continue to hold (qualitatively) even if we consider the setting in which the platform operates only when it can earn a profit. At the end of Section 4, we establish the robustness of our main results conditioning on the platform's profitability.

Sequence of events. Typically, the platform has stronger market power over other participants on the platform (Li and Wang 2024a). To capture this effect, we assume that the platform and

[7] https://gridwise.io/blog/delivery/gopuff-driver-pay-all-the-facts/

[8] https://www.moneylion.com/learn/how-much-does-instacart-pay/#:~:text=You%20will%20either%20be%20hired,or%20an%20in%2Dstore%20shopper

vendors move sequentially. Specifically, the sequence of events is as follows: In stage 1, the platform commits to a delivery fee p_d to charge customers. In stage 2, vendors simultaneously determine whether to participate on the platform and how much to charge customers for the food p_i^F (if participate) or full price p_i (if not participate). In Section 5.2, we revisit the system in which the platform and vendors move simultaneously. This extension pertains to the scenario in which the platform and vendors tend to hold equal market power. For instance, the increased activism from vendors or government intervention aimed at protecting small businesses may facilitate the conversation between vendors and the platform, thereby providing vendors with increased market power.

As a final remark, vendors may serve two customer channels: walk-in customers and delivery customers. In practice, vendors typically manage these channels separately and set different prices for each (Rana and Haddon 2023). Therefore, in our analysis, we assume that these two channels are operated independently by vendors and focus exclusively on the operations of the delivery channel.

3.2. Equilibrium Characterization

In this section, we describe the equilibrium characterization. We use $\chi_i = (J_i, p_i)$ to denote the strategy profile of vendor i, where $J_i \in \{0,1\}$ with $J_i = 1$ indicating vendor i participating on the platform, and $J_i = 0$ indicating vendor i building a dedicated delivery fleet. Then, the total number of vendors participating on the platform is given by $m = \sum_{i=1}^{n} J_i$, where $m \in \{0,\ldots,n\}$. Note that for notational simplicity, we use the full price p_i (i.e., food price p_i^F + delivery fee p_d) to describe vendor i's pricing strategy, regardless of whether vendor i participating on the platform. Given the delivery fee p_d charged by the platform, the actual decision (i.e., the food price) made by vendor i (given its participation) can be obtained directly from $p_i^F = p_i - p_d$. In addition, we do not explicitly include the decision of not operating in the delivery channel in vendor i's strategy profile, as this can be represented by either a sufficiently high food price (if $J_i = 1$) or a sufficiently high full price charged by vendor i (if $J_i = 0$). We use $\mathcal{P} = (\chi_1, \ldots, \chi_n)$ to denote the strategy profile of all vendors, and $\mathcal{P}_{-i} = \mathcal{P} \backslash \chi_i$ to denote the strategy profile of all vendors except for vendor i.

The best response strategy. Given the strategy profile $\mathcal{P}_{-i}$ of all other vendors, if vendor i chooses to build a dedicated delivery fleet, it decides on its full price p_i to maximize its profit. That is, vendor i solves the following problem:

$$\max_{p_i} \quad \lambda_i p_i - h \cdot \mu(\lambda_i; \overline{w}), \tag{D}$$

where, as defined in Section 3.1, $h \cdot \mu(\lambda_i; \overline{w})$ is the cost incurred to ensure the deliveries are made within the delivery window $\overline{w}$, given the realized demand rate λ_i. We use p_i^D and π_i^D to denote the optimal solution and optimal value, respectively, to Problem (D). If vendor i chooses to participate on the platform, it decides on its food price p_i^F to maximize its profit: $\max_{p_i^F} \quad (1-\gamma)\lambda_i p_i^F$. For convenience, we change the decision variable from p_i^F to $p_i = p_i^F + p_d$, where p_d is determined by the platform, and the problem solved by vendor i can be reformulated as

$$\max_{p_i} \quad (1-\gamma)\lambda_i(p_i - p_d). \tag{P}$$

We use p_i^P and π_i^P to denote the optimal solution and optimal value, respectively, to Problem (P). Vendor i chooses whichever option yields a higher profit. Specifically, the best response strategy χ_i^* of vendor i is given by:

$$\chi_i^* = \begin{cases} (1, p_i^P) & \text{if } \pi_i^P \geq \pi_i^D, \\ (0, p_i^D) & \text{if } \pi_i^P \leq \pi_i^D. \end{cases}$$

In Lemma 1, we characterize the set of best response strategies of a single vendor.

Lemma 1 (Best Response of a Single Vendor). *Given $\mathcal{P}_{-i}$, define*

$$M(\mathcal{P}_{-i}) = \theta + \beta \sum_{j \neq i}^{n} p_j, \tag{2}$$

and let

$$d(M) = M - \left(\frac{(M-h)^2 - 4h/\overline{w}}{1-\gamma} \right)^{\frac{1}{2}}. \tag{3}$$

(i) If $M(\mathcal{P}_{-i}) > h$ and $\overline{w} > 4h/[M(\mathcal{P}_{-i}) - h]^2$, the set of vendor i's best response strategies is given by

$$\mathcal{B}_i^*(\mathcal{P}_{-i}) = \begin{cases} \{(1, p_i^P)\} & \textit{if } p_d < d(M(\mathcal{P}_{-i})), \\ \{(0, p_i^D)\} & \textit{if } p_d > d(M(\mathcal{P}_{-i})), \\ \{(1, p_i^P), (0, p_i^D)\} & \textit{if } p_d = d(M(\mathcal{P}_{-i})), \end{cases}$$

where

$$p_i^P = \frac{M(\mathcal{P}_{-i}) + p_d}{2} \textit{ and } p_i^D = \frac{M(\mathcal{P}_{-i}) + h}{2}. \tag{4}$$

(ii) Otherwise, $(1, p_i^P) \in \mathcal{B}_i^*(\mathcal{P}_{-i})$.

In Lemma 1, $M(\mathcal{P}_{-i})$ represents the competitive market potential for vendor i. It denotes the demand rate that vendor i achieves if it provides the service for free, given the strategies of its competitors. Additionally, $d(M(\mathcal{P}_{-i}))$ represents the cutoff delivery fee: below this threshold, vendor i is better off participating on the platform, while above it, vendor i is better off building a dedicated delivery fleet. Consequently, Lemma 1 indicates that vendor i chooses to forgo the benefit of participating on the platform and remain independent by building a dedicated delivery fleet if it possesses sufficient competitive market potential (i.e., $M(\mathcal{P}_{-i}) > h$), customers exhibit a higher tolerance for delivery delays (i.e., $\overline{w} > 4h/[M(\mathcal{P}_{-i}) - h]^2$), and the platform's delivery fee is set too high (i.e., $p_d > d(M(\mathcal{P}_{-i}))$). Otherwise, vendor i chooses to participate on the platform.

Second-stage equilibrium. We refer to $\mathcal{P} = (\chi_1, \ldots, \chi_n)$ as a second-stage equilibrium profile if each vendor's strategy is its best response, meaning that no vendor has the incentive to deviate. Let $\Omega^*(p_d)$ denote the set of second-stage equilibrium profiles under the delivery fee p_d charged by the platform. Then we have

$$\Omega^*(p_d) = \left\{ \mathcal{P} \mid \chi_i \in \mathcal{B}_i^*(\mathcal{P}_{-i}), \quad \forall i \in \{1, \ldots, n\} \right\}, \tag{5}$$

where $\mathcal{B}_i^*(\mathcal{P}_{-i})$ is defined in Lemma 1. We characterize $\Omega^*(p_d)$ in details in Lemma A.1 of Online Appendix A.2. As demonstrated in Lemma A.1, a given p_d might induce multiple second-stage equilibrium profiles, leading to different numbers of vendors m participating on the platform. In order to analyze the full game between the platform and vendors, it is imperative to predict how vendors respond to the platform's strategy. Hence, we refine the game among vendors using the following refinement rule.

Remark (Refinement Rule)**.** If the platform's strategy p_d leads to multiple second-stage equilibria with either $m = 0$ (indicating no vendors participating on the platform) or $m > 0$ (suggesting the platform has a positive market share), we prioritize the ones with $m > 0$. Among multiple second-stage equilibria in which the platform has a positive market share, we select the one that maximizes the platform's profit. As demonstrated in Online Appendix A.2, this refinement rule aligns with the selection of the second-stage equilibrium with the largest number of vendors participating on the platform. □

The aforementioned refinement rule serves the platform's interests best in terms of profit. This refinement rule is consistent with the practice: given the stronger market power of the platform, it can actively shape vendor participation and steer the system evolution in its desired way. For example, Uber Eats actively delists its partnered restaurants that fail to meet its specifications (Rana and Haddon 2023), and Grubhub once offered a $250 incentive to encourage the participation of restaurants (Splitter 2020).[9]

Equilibrium. The platform decides on the delivery fee p_d to charge customers to maximize its

[9] The latter cash incentive is a one-time compensation the platform offers to entice vendor participation, which does not change the platform's long-run profit rate. Nevertheless, it can serve as a lever for the platform to shape the system's evolution during its early stages.

profit. Recall that the platform shares a fraction γ of revenues from vendors participating on the platform. Then, the problem solved by the platform can be formulated as

$$\max_{p_d} \quad \Pi(p_d) = \sum_{i=1}^{n} \lambda_i \left(\gamma p_i^F + p_d\right) \cdot J_i - h \cdot \mu(\sum_{i=1}^{n} \lambda_i \cdot J_i; \overline{w}) \qquad \text{(Platform Problem)}$$

$$\text{subject to} \quad p_d \in \{p_d' \mid \Omega^*(p_d') \neq \emptyset\},$$

$$\mathcal{P} \in \Omega^*(p_d),$$

$$0 < m = \sum_{i=1}^{n} J_i \in \{1, \ldots, n\}, \qquad \text{(Engagement Constraint)}$$

where $J_i \in \{0,1\}$ is the indicator denoting whether vendor i participates on the platform, and $h \cdot \mu(\sum_{i=1}^{n} \lambda_i \cdot J_i; \overline{w})$ is the cost incurred to ensure the delivery time is within the delivery window $\overline{w}$, given that aggregate demand rate associated with the platform $\sum_{i=1}^{n} \lambda_i \cdot J_i$. The first constraint specifies that we confine our analysis to the platform's strategies in which the delivery fee p_d leads to a non-empty set of second-stage equilibria, i.e., $\Omega^*(p_d) \neq \emptyset$, where $\Omega^*(p_d)$ is defined in (5). As demonstrated in Lemma A.1, a delivery fee p_d charged by the platform may induce a unique second-stage equilibrium, multiple ones, or none at all (the last scenario occurs only if a condition, which we will refer to as the "Subsidy Condition" later in this section, is not satisfied). We restrict our attention in the first stage to scenarios in which a second-stage equilibrium exists. Practically, the absence of a second-stage equilibrium suggests a less predictable market outcome, posing a greater risk for the platform when operating in such a market. Therefore, the platform may avoid setting delivery fees that induce the absence of a second-stage equilibrium.

The Engagement Constraint ensures the market presence of the platform. As stated earlier in Section 3.1, maintaining market presence often takes precedence over generating profit for a startup platform. In addition, setting aside the profitability of the platform allows us to gain a clear understanding of how the competition among vendors is impacted by the introduction of the platform. At the end of Section 4, we examine the profitability of the platform under the optimal solution to the Platform Problem, and qualitatively establish the robustness of our results if the Engagement Constraint is relaxed (see Lemma 2).

Let p_d^* denote the delivery fee charged by the platform, and m^* denote the number of vendors participating on the platform in equilibrium. We use the superscript P to describe outcomes for vendors participating on the platform and the superscript D to describe outcomes for vendors building dedicated delivery fleets. Define

$$p^P(m, p_d) = \frac{\theta + p_d + \beta(n-m)(h-p_d)/(2+\beta)}{2 - \beta(n-1)}, \tag{6}$$

$$p^D(m,p_d) = \frac{\theta + p_d + [2-\beta(m-1)](h-p_d)/(2+\beta)}{2-\beta(n-1)}, \quad \text{and} \tag{7}$$

$$p^F(m,p_d) = p^P(m,p_d) - p_d = \frac{\theta + \beta(n-m)(h-p_d)/(2+\beta) - [1-\beta(n-1)]p_d}{2-\beta(n-1)}. \tag{8}$$

We say that the platform *subsidizes* the delivery service per order if it offers a delivery fee below the marginal cost per unit of demand, i.e., $p_d^* < h$, and it *profits* per order from delivery service otherwise. The following proposition characterizes the equilibrium.

Proposition 1 (Equilibrium). *(i) If*

$$\frac{h}{\overline{w}} < \gamma \left(\frac{\theta - [1-\beta(n-1)]h}{2-\beta(n-1)} \right)^2 \qquad \text{(Subsidy Condition)}$$

is satisfied, there exists a unique equilibrium under which the platform subsidizes the delivery service per order, i.e., $p_d^* < h$, and all vendors participate on the platform, i.e., $m^* = n$.

(ii) Otherwise, the platform profits per order from delivery service, i.e., $p_d^* \geq h$, and the equilibrium can be asymmetric with some vendors participating on the platform and the others building dedicated delivery fleets, i.e., $m^* \leq n$.

Moreover, in equilibrium, vendors participating on the platform charge customers a food price $p^F(m^*, p_d^*)$, and customers pay a full price $p^P(m^*, p_d^*)$ per order; and those building dedicated delivery fleets charge customers a full price $p^D(m^*, p_d^*)$ per order.

Proposition 1 states that if the Subsidy Condition is satisfied, the platform employs strategies of subsidizing the delivery service per order, resulting in the participation of all vendors in equilibrium. Otherwise, the platform profits per order from the delivery service, and the equilibrium outcome can be asymmetric. Specifically, there are cases in which some vendors choose to participate on the platform while others establish their own dedicated delivery fleets. In practice, both per-order delivery subsidy and per-order delivery profiting may be observed. For instance, Grubhub once provided subsidized delivery fees to its partnered restaurants to offer them support (Forman 2020), while Uber Eats previously modified its fee structure by introducing a service fee and a small-order fee (Liao 2019), potentially resulting in higher delivery fees and the delivery profiting per order.

Observe that the Subsidy Condition is more likely to be satisfied if the delivery window is lengthy, i.e., $\overline{w}$ is large, the marginal cost h is low, or the market potential θ is large. In these scenarios, building a dedicated delivery fleet is more feasible for vendors, as either the cost of maintaining the service standard is low or the scale of the vendor is sufficiently large to support the establishment of a dedicated delivery fleet. Therefore, to recruit vendors, the platform needs to subsidize the delivery service per order to enhance its attractiveness.

Conversely, if the delivery window is short, i.e., $\overline{w}$ is small, the marginal delivery cost h is high, or the market potential θ is small, it is more likely that participating on the platform is preferred by vendors. This could be either to avoid the significant cost of maintaining the service standard or because the market potential is too thin to support the establishment of a dedicated delivery fleet. This allows the platform to leverage economies of scale and impose a higher delivery fee while still ensuring the participation of some vendors. The platform's strategy also shifts from attracting vendors to generating profits from per-order delivery service, which may lead to the departure of some vendors from the platform and lead to an asymmetric equilibrium outcome.

Note that the platform has two revenue streams: one from revenue sharing, i.e., a fraction γ of each vendor's revenue, and the other from the per-order delivery fee p_d, where the delivery fee p_d charged by the platform also influences the prices charged by vendors and, consequently, the total prices paid by customers. The platform achieves a positive profit if the combined revenue from these two streams is sufficient to offset the cost of building service capacity, denoted by $h \cdot \mu(\sum_{i=1}^{n} \lambda_i \cdot J_i; \overline{w})$. Therefore, on the one hand, even when the platform subsidizes the delivery service per order, it can still be profitable as the revenue from the revenue-sharing stream can compensate for the per-order delivery subsidy. On the other hand, the platform may not necessarily be profitable, even when it profits per order from the delivery service, if the costs associated with building service capacity to maintain the desired service level are excessively high.

4. Impact of Platform Entry

In this section, we characterize the impact of introducing the delivery platform on vendor competition. We do so by comparing the equilibrium outcomes in systems with and without the platform in terms of vendor profits and customer surplus (to be formally defined later in Section 4.2). We analyze the system without the platform in Section 4.1 and compare the equilibrium outcomes in Section 4.2.

4.1. System Without The Platform

In the system without the platform, each vendor decides on its full price p_i to charge customers to maximize its profit (while the decision of not operating in the delivery channel can be captured by a high full price). That is, all vendors solve problem (D) simultaneously. In the following proposition, we characterize the equilibrium outcome for the system without the platform.

Proposition 2 (Without The Platform)**.** *In the system without the platform, the equilibrium outcome is characterized as follows.*

(i) If

$$\theta > [2-\beta(n-1)]\sqrt{\frac{h}{\overline{w}}}+[1-\beta(n-1)]h, \qquad \text{(Vendor Viability Condition)}$$

is satisfied, there exists a unique equilibrium under which all vendors charge customers a full price

$$p^N = \frac{\theta + h}{2-\beta(n-1)}. \tag{9}$$

(ii) Otherwise, there does not exist an equilibrium under which all vendors can make positive profits. Moreover, when $\theta \le 2[1-\beta(n-1)]\sqrt{h/\overline{w}}+[1-\beta(n-1)]h$, there exists an equilibrium in which no vendors choose to operate in the delivery channel.

The Vendor Viability Condition states that the market potential for vendors is sufficiently large to support them to make positive profits by establishing dedicated delivery fleets and meeting the delivery window constraint. When the Vendor Viability Condition is not satisfied, there does not exist an equilibrium in which all vendors can be profitable. Furthermore, when the Vendor Viability Condition is further violated such that $\theta \le 2[1-\beta(n-1)]\sqrt{h/\overline{w}}+[1-\beta(n-1)]h$ ($<$ R.H.S. of the Vendor Viability Condition), there exists an equilibrium in which no vendors choose to operate, corresponding to a scenario in which vendors choose not to provide the delivery option (i.e., only catering to the walk-in channel of customers).

4.2. Compare Systems With and Without The Platform

In this section, we compare systems with and without the platform. In light of Proposition 2, when the Vendor Viability Condition is not satisfied, there does not exist an equilibrium in which all vendors make profits from building dedicated delivery fleets. Furthermore, an equilibrium can arise in which none of the vendors choose to operate in the delivery channel. In this case, introducing the platform benefits both vendors and customers by reducing barriers for vendors who otherwise do not offer delivery services, thereby making them more accessible to customers. In the remainder of this section, we restrict our attention to the case in which building a dedicated delivery fleet is a viable option for all vendors. That is, we focus on cases in which the Vendor Viability Condition is satisfied.

Competition intensity and customer surplus. In view of (6), (7) and (9), p^P, p^D, and p^N denote the prices that customers pay when vendors participate on the platform, build a dedicated delivery fleet in the presence of the platform, and in the system without the platform, respectively. Correspondingly, for these scenarios, we define π^P, π^D, and π^N as vendor profits and λ^P, λ^D, and λ^N as customer demand rates. Additionally, we introduce the parameters u^P and u^N to denote the individual customer surplus in systems with and without the platform. The entry of the delivery platform has the potential to reshape the competitive landscape in the market by attracting vendors to participate and influencing their pricing strategies. This can result in either intensified or alleviated competition. In the following definition, we formally define the impact of the introduction of the delivery platform on vendor competition and individual customer surplus.

Definition 1 (Competition Intensity and Customer Surplus)**.** *We use*

(i) $u^P \succ u^N$ *to denote that the introduction of the platform intensifies vendor competition and increases individual customer surplus, which is equivalent to* $p^P < p^N$, $\lambda^P > \lambda^N$, $p^D < p^N$ *and* $\lambda^D > \lambda^N$, *and*

(ii) $u^P \preceq u^N$ *to denote that the introduction of the platform alleviates vendor competition and decreases individual customer surplus, which is equivalent to* $p^P \geq p^N$, $\lambda^P \leq \lambda^N$ $p^D \geq p^N$ *and* $\lambda^D \leq \lambda^N$*.*

In the first scenario of Definition 1 (i.e., $u^P \succ u^N$), the introduction of the platform encourages all vendors, regardless of whether they participate on the platform, to engage in more aggressive competition by adopting strategies that result in lower overall prices for customers, thereby stimulating demand (i.e., $p^P < p^N$, $\lambda^P > \lambda^N$, $p^D < p^N$ and $\lambda^D > \lambda^N$). In this case, we refer to the introduction of the platform as intensifying vendor competition. Moreover, as it implies that all vendors fulfill more customer demands, with each customer paying less, we say the individual customer surplus is higher. Conversely, in the second scenario (i.e., $u^P \preceq u^N$), the introduction of the platform allows all vendors, regardless of whether they participate on the platform, to select a strategy that raises the overall prices for customers and forgo some demand (i.e., $p^P \geq p^N$, $\lambda^P \leq \lambda^N$ $p^D \geq p^N$ and $\lambda^D \leq \lambda^N$). In this case, we refer to the introduction of the platform as alleviating vendor competition. As it implies that all vendors fulfill fewer customer demands, with each customer paying more, we say the individual customer surplus is lower.

Impact of platform entry. In the following proposition, we compare systems with and without the platform with respect to vendor profits and individual customer surplus.

Proposition 3 (Impact of Platform Entry)**.** *Compared with the system without the platform,*

(i) if the Subsidy Condition *is satisfied, the introduction of the platform intensifies vendor competition and increases individual customer surplus i.e.,* $u^P \succ u^N$, *leading to lower vendor profits, i.e.,* $\pi^P \leq \pi^N$ *(in this case all vendors participate on the platform by Proposition 1, i.e.,* $m^* = n$*);*

(ii) otherwise, the introduction of the platform (weakly) alleviates vendor competition and (weakly) decreases individual customer surplus, i.e., $u^P \preceq u^N$, *leading to (weakly) higher vendor profits, i.e.,* $\pi^P \geq \pi^N$ *and* $\pi^D \geq \pi^N$*.*

Proposition 3 uncovers a nuanced impact that arises from the introduction of the platform. Perhaps paradoxically, relative to the system without the platform, introducing the platform hurts vendors when the platform subsidizes the delivery service per order; and it benefits vendors (regardless of whether they participate on the platform) when the platform profits per order from the delivery service. As we shall discuss next, the results are attributed to either intensified or alleviated competition by introducing the platform.

By subsidizing the delivery service per order (i.e., $p_d^* < h$), the platform reduces operational costs and offers a competitive edge to vendors who participate on the platform. This intensifies the competition among vendors by encouraging them to adopt a strategy that lowers the overall price for customers (i.e., $p^P < p^N$) to attract more demand (i.e., $\lambda^P > \lambda^N$), leading to an increase in individual customer surplus (i.e., $u^P \succ u^N$). Consequently, the introduction of the platform lowers vendor profits (i.e., $\pi^P < \pi^N$) by compelling them to engage in more intensified competition.

In contrast, when the platform profits per order from the delivery service (i.e., $p_d^* < h$) in addition to revenue sharing, it raises the operational costs and reduces the competitive edges for vendors who choose to participate on the platform. Consequently, those vendors refrain from engaging in intense competition and instead adopt strategies that raise the overall price for customers (i.e., $p^P \geq p^N$) and actively forgo some demand (i.e., $\lambda^P \leq \lambda^N$). As a spillover effect, vendors who decide not to participate on the platform also increase their full prices (i.e., $p^D \geq p^N$) and cater to fewer customers (i.e., $\lambda^D \leq \lambda^N$). Therefore, the introduction of the platform alleviates vendor competition and increases individual customer surplus (i.e., $u^P \preceq u^D$), and vendors, whether participating on the platform or building dedicated delivery fleets, make more profits (i.e., $\pi^P \geq \pi^N$ and $\pi^D \geq \pi^N$). A related observation to our findings in the context of e-commerce is documented by Mitchell (2024): Amazon may prevent competition to maintain high retail prices by imposing high fees on its third-party sellers and punishing them by listing cheaper elsewhere.

Notably, when the Subsidy Condition is satisfied, the platform subsidizes the delivery service per order in equilibrium to enhance its attractiveness and encourage vendor participation. However, this seemingly appealing per-order delivery subsidy (offered by the platform to attract vendor participation) hurts vendors by compelling them to engage in more intensive competition, ultimately eroding their profits. Consequently, the platform-enabled sharing of couriers improves delivery efficiency and lowers delivery costs but may intensify vendor competition through per-order delivery subsidy, placing them in a prisoner's dilemma that ultimately lowers their profits.

In this work, although we investigate an equilibrium outcome, our results can be viewed as the stationary point evolved out of a dynamic process. For example, under the Subsidy Condition, the platform starts by subsidizing per-order delivery for some vendors to attract their participation. This initial aggregation of demand and supply enables the platform to operate with a lower average delivery cost per unit of demand due to economies of scale while also intensifying vendor competition and increasing the volume. The increased volume further enhances the platform's economies of scale, leading to an even lower average delivery cost. This, in turn, gives the platform greater potential to subsidize the delivery service per order, attracting more vendors and creating a flywheel effect until all vendors participate on the platform, resulting in a prisoner's dilemma for vendors and potential profitability for the platform.

Profitability of the platform. As discussed in Section 3.2, maintaining market presence often takes precedence over generating profit for a startup. Therefore, in our base model, we solve the Platform Problem under the Engagement Constraint. Nevertheless, we demonstrate that Propositions 1 and 3 remain valid even when conditioning on the platform's profitability. To that end, we introduce the following lemma, which characterizes the set of parameters that ensure that the platform achieves profitability in equilibrium.

Lemma 2 (Potential Platform Profitability). *Define the set of parameters* $\Gamma \triangleq \left\{(\overline{w}, h, \theta) \mid h/\overline{w} = \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2 \text{ and the Vendor Viability Condition is satisfied}\right\}$. *There exists* $\epsilon > 0$ *such that the platform makes a positive profit in equilibrium, i.e.,* $\Pi(p_d^*) > 0$, *if* $d(\Gamma, (\overline{w}, h, \theta)) \triangleq \sqrt{(\overline{w}-\overline{w}')^2 + (h-h')^2 + (\theta-\theta')^2} \le \epsilon$ *for all* $(\overline{w}', h', \theta') \in \Gamma$.

In Lemma 2, $d(\Gamma, (\overline{w}, h, \theta))$ represents the distance between $(\overline{w}, h, \theta)$ and the parameter set Γ. Then, Lemma 2 says that the platform is profitable in equilibrium, i.e., $\Pi(p_d^*) > 0$, when the system parameters $(\overline{w}, h, \theta)$ are sufficiently "close" to the parameter set Γ. It follows that given any point $(\overline{w}', h', \theta') \in \Gamma$, there exists a neighborhood of $(\overline{w}', h', \theta')$ such that $\Pi(p_d^*) > 0$. This neighborhood must include two types of parameters: one under which the Subsidy Condition is satisfied, resulting in the platform subsidizing per-order delivery service, thereby intensifying vendor competition, lowering vendor profits, and increasing individual customer surplus, and the other under which the Subsidy Condition is unsatisfied, allowing the platform to profit per order from the delivery service, thereby alleviating vendor competition, increasing vendor profits, and decreasing individual customer surplus. Figure 1 illustrates the profitability parameter region for the platform, showing where per-order delivery subsidy and per-order delivery profiting occur (conditional on the platform's profitability) in a system with $n = 30$ vendors, a commission rate $\gamma = 0.2$, and a cross-price sensitivity $\beta = 0.031$.

Finally, we note that the setting we consider in the paper is the one in which vendors incur a 0 commission to the platform when they choose to build dedicated delivery fleets. This corresponds

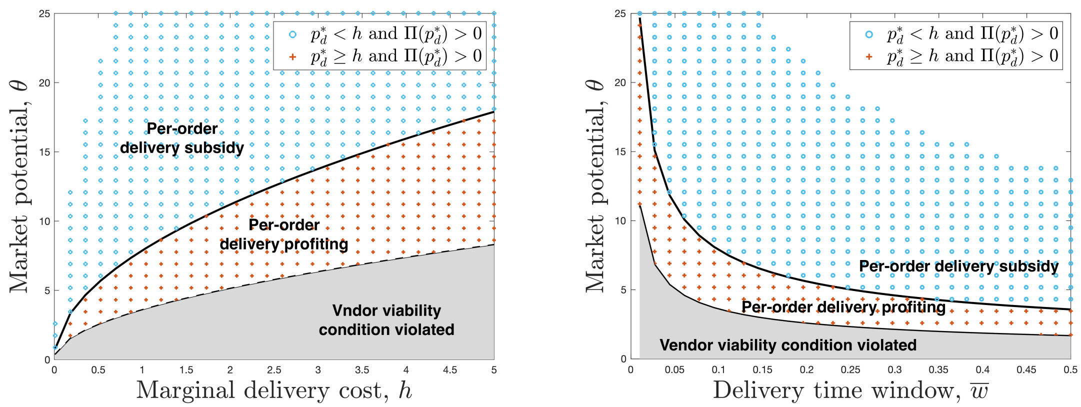

Figure 1 **Illustration of the profitability parameter region for the platform (left) on (θ, h) given $\overline{w} = 0.1$ and (right) on $(\theta, \overline{w})$ given $h = 1$**

to vendors establishing their own in-house delivery. In practice, there are cases where vendors can choose to be listed on the platform while using their own dedicated couriers (i.e., Doordash and Uber Eats have such a program[10]), paying the platform a lower (but positive) commission rate. In such scenarios, our main findings and insights remain applicable. In this case, the cost associated with vendors building their own delivery fleet would be higher, making them more likely to use the platform's delivery service. Consequently, the platform's need to subsidize the delivery service per order (which intensifies vendor competition) would decrease. Nevertheless, our insights regarding alleviating versus intensifying competition would still hold, albeit under slightly different conditions.

5. Extensions and Discussions

In this section, we expand our analysis with a set of extensions. These include investigating how the level of substitutability of vendors influences the outcome resulting from the introduction of the platform (Section 5.1), exploring an alternative decision-making process that accounts for a different level of market power between the platform and vendors (Section 5.2), examining systems under alternative contracts between the platform and vendors (Section 5.3), considering the presence of asymmetric vendor scales (Section 5.4), and studying the system in which couriers are paid based on piecemeal delivery per order (Section 5.5).

[10] See `https://get.doordash.com/en-us/products/self-delivery` and `https://help.uber.com/en/merchants-and-restaurants/article/using-your-own-delivery-staff?nodeId=a37aee35-1dac-4509-ac8c-28c6aefbf265`

5.1. The Role of Substitutability

In this section, we investigate how the level of substitutability among vendors influences the effects of introducing a delivery platform on vendor competition. We modify our model based on a commonly used demand system for multi-firm competition with product differentiation. In particular, we specify the demand rate associated with vendor i by

$$\lambda_i = \left(\frac{\theta}{1+\delta} - \frac{p_i}{1-\delta^2} + \frac{\delta \sum_{j\neq i} p_j}{(n-1)(1-\delta^2)} \right)^+, \tag{10}$$

where $\delta \in [0,1)$. The parameter δ can measure the level of substitutability among vendors, with a larger δ implying a higher cross-price sensitivity (and $\delta = 0$ indicating each vendor is a local monopoly). When the market is more homogeneous, i.e., δ is larger, one expects a smaller aggregate market demand given the same prices and lower equilibrium prices of competing vendors due to less differentiated substitutes. Therefore, in contrast to using (1) to specify the demand rate, when investigating the role of vendor substitutability, (10) offers advantages as follows. First, provided that all vendors have positive market shares, i.e., $\lambda_i > 0$ for all $i \in \{1, \dots, n\}$, (10) guarantees the aggregate demand rate of the whole market $\sum_{i=1}^{n} \lambda_i = (n\theta - \sum_{i=1}^{n} p_i)/(1+\delta)$ is decreasing in δ; and second, (10) ensures that the equilibrium price without the platform control $p^N = 1-(1-h)(2-\delta)$ is decreasing in δ as expected (see Federgruen and Hu 2016 and McGuire and Staelin 2008 for more discussions therein). With these desired properties, (10) offers an appropriate micro foundation for analyzing the level of substitutability compared to (1).

Notice that the analysis under the demand structure specified by (1) is perfectly valid when the level of substitutability among vendors is fixed (i.e., $\alpha = 1$ and β being fixed). We modify the demand structure to (10) only for the purpose of investigating the role of substitutability. We compare systems with and without the platform under the demand structure specified as (10). The following proposition presents results that parallel those obtained in Propositions 1 and 3.

Proposition 4 (Substitutability)**.** *Suppose that the demand structure is specified as (10). For any model parameters satisfying $h/\overline{w} < \gamma(\theta-h)^2/4$, there exists a threshold on the level of substitutability $\hat{\delta} \in (0,1)$, such that:*

(i) If $h/\overline{w} < \gamma(\theta-h)^2/4$ and $\delta \in [0, \hat{\delta})$, the platform subsidizes the delivery service per order in equilibrium, i.e., $p_d^* < h$. Then the introduction of the platform intensifies vendor competition and increases individual customer surplus, i.e., $u^P \succ u^N$, leading to lower vendor profits, i.e., $\pi^P < \pi^N$ (in this case all vendors participate on the platform in equilibrium, i.e., $m^* = n$);

(ii) Otherwise, i.e., $h/\overline{w} \geq \gamma(\theta-h)^2/4$, or $h/\overline{w} < \gamma(\theta-h)^2/4$ and $\delta \in [\hat{\delta}, 1)$, the platform profits per order from the delivery service in equilibrium, i.e., $p_d^* \geq h$. Then the introduction of the

platform (weakly) alleviates vendor competition and (weakly) decreases individual customer surplus, i.e., $u^P \preceq u^N$, leading to (weakly) higher vendor profits, i.e., $\pi^P \geq \pi^N$ and $\pi^D \geq \pi^N$.

Proposition 4 states that if the delivery window $\overline{w}$ is not too short, the marginal delivery cost h is not too high, or the market potential θ is not too small, such that $h/\overline{w} < \gamma(\theta - h)^2/4$, the effect of introducing the platform on vendor competition can vary based on the level of substitutability among vendors. In particular, there exists a threshold on the level of substitutability $\hat{\delta}$ such that if $\delta < \hat{\delta}$, the introduction of the platform intensifies vendor competition, leading to decreases in vendor profits and increases in individual customer surplus; and if $\delta \geq \hat{\delta}$, the introduction of the platform alleviates vendor competition, leading to increases in vendor profits and decreases in individual customer surplus. The reasons behind these results are similar to those of the main model. With a high level of substitutability (i.e., $\delta \geq \hat{\delta}$), vendors are already operating in an intensely competitive environment, earning low profits in the system without the platform. In this case, attracting vendors is relatively easier, enabling the platform to profit per order from the delivery service while still maintaining its market presence, with some vendors participating on the platform in equilibrium (i.e., $m^* > 0$). This raises the operational cost for vendors who participate on the platform, leading them to be reluctant to compete aggressively, as previously explained. Due to a spillover effect, vendors who build a dedicated delivery fleet also refrain from intensive competition by increasing their full prices. The reverse is true when the level of substitutability is low (i.e., $\delta < \hat{\delta}$).

Otherwise, i.e., $h/\overline{w} \geq \gamma(\theta - h)^2/4$, regardless of the level of substitutability δ, the platform profits per order from the delivery service and alleviates vendor competition upon its introduction, which leads to increases in vendor profits and decreases in individual customer surplus as previously explained.

5.2. Simultaneous Movement by The Platform and Vendors

In our base model, we consider a setting in which the platform and vendors move sequentially, with the platform first determining the delivery fee p_d, followed by vendors deciding on whether to participate on the platform and how much to charge customers for food or full prices. In this section, we extend our analysis by considering the setting in which the platform and vendors move simultaneously. We note that the decision-making order (sequential or simultaneous) reflects how much market power the platform and vendors have relative to each other. When the platform and vendors move sequentially, the platform holds more power than the vendors (akin to a leader in a leader-follower game). In contrast, we use simultaneous movement to capture the scenario in which the platform and vendors tend to have equal market power.

When the platform and vendors move simultaneously, the vendor's strategy profile and the platform's strategy $(\mathcal{P}, p_d)$ form an equilibrium if and only if

$$\mathcal{P} \in \Omega^*(p_d) \quad \text{and} \quad p_d \in \arg\max \Pi(p_d), \tag{11}$$

where $\Omega^*(p_d)$ is the set of vendor equilibrium strategies given p_d as defined in (5) and $\Pi(p_d)$ denote the platform's profit as a function of p_d defined in the Platform Problem. In Online Appendix C.2, we solve Problem (11) and characterize the necessary and sufficient condition for the existence of an equilibrium with m vendors participating on the platform (see Proposition C.1).

In contrast to the findings of Propositions 1 and 3, the following proposition demonstrates that when the platform and vendors move simultaneously, the introduction of the platform does not intensify vendor competition. Consequently, it does not harm vendors but hurts customers.

Proposition 5 (Simultaneous Move)**.** *When the platform and vendors move simultaneously, either an equilibrium with a positive number of vendors participating on the platform does not exist, or the platform generates profit per order from the delivery service in equilibrium, i.e., $p_d^* > h$. In the latter case, the introduction of the platform alleviates vendor competition, i.e., $u^P \prec u^N$, leading to higher vendor profits, i.e., $\pi^P > \pi^N$ and $\pi^D > \pi^N$.*

Proposition 5 reveals that when the platform and vendors move simultaneously, its introduction does not intensify vendor competition but positively impacts vendors. Compared to the case in which the platform and vendors move sequentially, the platform has less control over the vendor strategies (i.e., the platform does not have a relatively stronger power) when they move simultaneously than in the simultaneous movement game. Therefore, with the introduction of the platform, it is hard to form an equilibrium in which vendors are worse off.

As mentioned in Section 3.1, in practice, the market power resides predominantly with the platform. However, with increased activism from vendors or government intervention aimed at protecting small businesses, vendors are likely to gain more market power and, consequently, less likely to be harmed by the introduction of the platform. Then the existence of the platform creates a win-win scenario for both the platform and the vendors.

5.3. Alternative Contracts

In our base model, we consider the setting in which vendors participating on the platform share a fixed fraction $\gamma \in (0, 1)$ of their revenue with the platform. We refer to this agreement between the platform and vendors as a *revenue-sharing contract*. In practice, there may be some variations of the revenue-sharing contract. For instance, the platform may choose to offer a fixed participation fee to reward vendors for their participation (see Splitter 2020 for an example). Furthermore, the platform might impose a per-order transaction fee on vendors. In a specific scenario in which the platform does not share the vendors' revenues but charges vendors a per-order transaction fee, we term the arrangement between the platform and vendors a *transaction-based* contract (e.g., the DoorDash Drive program allows merchants with their own apps or websites to access the DoorDash delivery service by paying a fixed delivery fee per order[11]). To encompass these variations, we introduce a generalized contract form, which we refer to as the general contract, as follows. For vendors participating on the platform, the platform charges a transaction fee of p_t per delivery and their customers a delivery fee of p_d. Additionally, vendors share a fraction γ of their revenues (i.e., their food prices p_i^F) with the platform, and the platform pays them a participation reward rate B.

Under the general contract, vendor i solves Problem (D) by building a dedicated delivery fleet, and it solves the following problem by participating on the platform:

$$\max_{p_i} \quad \lambda_i \left[(1-\gamma)(p_i - p_d) - p_t\right] + B. \tag{12}$$

Abusing a bit notation, under the general contract, we use $\Omega^*(p_d, p_t)$ to denote the set of second-stage equilibrium profiles of vendors given the delivery fee p_d to charge customers and the transition fee p_t to charge vendors by the platform (note that $\Omega^*(p_d, p_t)$ is an analog to the definition of $\Omega^*(p_d)$ in (5)). The platform decides on the delivery fee and the transaction fee to maximize its profit:

$$\begin{aligned} \max_{p_d, p_t} \quad & \Pi(p_d, p_t) = \sum_{i=1}^{n} \lambda_i \left(\gamma p_i^F + p_d + p_t\right) \cdot J_i - h \cdot \mu\left(w_p, \sum_{i=1}^{n} \lambda_i \cdot J_i\right) - B \sum_{i=1}^{n} \lambda_i \cdot J_i, \qquad (13) \\ \text{subject to} \quad & (p_d, p_t) \in \{(p_d', p_t') \mid \Omega^*(p_d', p_t') \neq \emptyset\}, \\ & \mathcal{P} \in \Omega^*(p_d, p_t), \\ & m = \sum_{i=1}^{n} J_i \in \{1, \ldots, n\}. \end{aligned}$$

Notice that most contracts adopted in practice are special cases of the general contract we introduced. For instance, if we set $p_t = 0$ and $B = 0$, the general contract simplifies to a revenue-sharing contract. Similarly, if $\gamma = 0$ and $B = 0$, it becomes a transaction-based contract. The following proposition provides results akin to those in Propositions 1 and 3.

Proposition 6 (General Contract)**.** *Under the general contract:*

(i) If

$$\frac{h}{w} + B < \gamma \left(\frac{\theta - [1 - \beta(n-1)]h}{2 - \beta(n-1)}\right)^2 \tag{14}$$

[11] https://get.doordash.com/en-ca/learning-center/delivery-commission

is satisfied, the platform charges low delivery and transaction fees in equilibrium such that $p_d^* + \frac{p_t^*}{1-\gamma} < h$*. Then the introduction of the platform intensifies the vendor competition and increases individual customer surplus, i.e.,* $u^P \succ u^N$*, leading to lower vendor profits, i.e.,* $\pi^P < \pi^D$ *(in this case all vendors participate on the platform in equilibrium, i.e.,* $m^* = n$*).*

(ii) Otherwise, the platform charges high delivery and transaction fees in equilibrium such that $p_d^* + \frac{p_t^*}{1-\gamma} \geq h$*. Then the introduction of the platform (weakly) alleviates the vendor competition and (weakly) decreases individual customer surplus, i.e.,* $u^P \preceq u^N$*, leading to (weakly) higher vendor profits, i.e.,* $\pi^P \geq \pi^N$ *and* $\pi^D \geq \pi^N$*.*

By Proposition 6, under the general contract, the core impact of introducing the platform on vendor competition always hinges on the delivery and transaction fees it charges in equilibrium. Specifically, the introduction of the platform intensifies vendor competition, leading to lower vendor profits and higher individual customer surplus if the platform charges low delivery and transaction fees, i.e., $p_d^* + \frac{p_t^*}{1-\gamma} < h$, and it alleviates vendor competition, leading to higher vendor profits and lower individual customer surplus otherwise, i.e., $p_d^* + \frac{p_t^*}{1-\gamma} \geq h$. Note that, under the general contract, the condition for the introduction of the platform to intensify vendor competition, i.e., $p_d + \frac{p_t}{1-\gamma} < h$ is more stringent than the per-order deliver subsidy, i.e., $p_d + p_t < h$. This is because, given vendor i's margin $(1-\gamma)(p_i - p_d) + p_t$, the decrease in profit margin due to an increase in the delivery fee is shared by the platform through the commission, whereas the transaction fee p_t directly impacts the vendor without the mitigation by the platform.

By (14), it is possible to design contracts by, for example, regulating the commission rate γ and the participation award B to steer the platform's choices towards the desired direction that either favors customers (i.e., (14) being satisfied) or vendors (i.e., (14) being violated). For instance, under the transaction-based contract (i.e., $\gamma = 0$ and $B = 0$), regardless of the system characteristics, the introduction of the platform always alleviates vendor competition i.e., (14) is always violated. This suggests that moving towards the transaction-based contract could avoid the prisoner's dilemma in which vendors are harmed by the platform entry. This occurs because, under the transaction-based contract, vendors only incur the cost per delivery (rather than share a fraction of revenue with the platform, as is the case under the revenue-sharing contract) to be exempt from building service capacity themselves. Consequently, vendors are more tolerant of higher delivery and transaction fees. This, following the same logic as discussed in Proposition 3, raises the vendors' operational costs, alleviating the competition intensity among vendors.

5.4. Systems with Asymmetric Vendors

In our base model, we consider symmetrical demand functions among all vendors. In practice, vendors may vary in their scales, which affects the demand they can cater to and their decisions to participate on the platform. In this section, we expand our analysis to model vendors of various scales. Let vendor i's demand rate be

$$\lambda_i = s_i \left(\theta - p_i + \beta \sum_{j \neq i} p_j \right)^+,$$

where $0 < s_1 \leq s_2 \leq \cdots \leq s_n$ without lose of generality. Similar expressions for the demand rate are utilized in McGuire and Staelin (2008). We then consider the same problem as outlined in our base model. Specifically, in stage 1, the platform commits to a delivery fee p_d to charge customers; and in stage 2, vendors simultaneously determine whether to participate on the platform and how much to charge customers for the food p_i^F (if they participate) or full price p_i (if they do not participate). Then the following proposition serves as a counterpart to Propositions 1 and 3 from our base model.

Proposition 7 (Asymmetric Vendor Scales)**.** *In the system with asymmetric vendor scales:*

(i) If $h/(s_1\overline{w}) < \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, the platform subsidizes the delivery service per order in equilibrium, i.e., $p_d^* < h$. The introduction of the platform intensifies vendor competition and increases individual customer surplus, i.e., $u^P \succ u^D$. There exists $m^* \in \{1, \ldots, n\}$ such that smaller vendors $\{1, \ldots, m^*\}$ participate on the platform and larger vendors $\{m^* + 1, \ldots, n\}$ build dedicated delivery fleets. Moreover, there exists $k < m^*$ such that with the platform, the profits for smaller vendors $\{1, \ldots, k\}$ are higher, i.e., $\pi_i^P > \pi_i^N$, and those for larger vendors $\{k+1, \ldots, n\}$ are lower, i.e., $\pi_i^P < \pi_i^N$ for $i \in \{k+1, \ldots, m^*\}$ and $\pi_i^D < \pi_i^N$ for $i \in \{m^* + 1, \ldots, n\}$.

(ii) If $h/(s_n\overline{w}) \geq \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, the platform profits per order from the delivery service in equilibrium, i.e., $p_d^* \geq h$. The introduction of the platform (weakly) alleviates vendor competition and (weakly) decreases individual customer surplus, i.e., $u^P \preceq u^N$. Moreover, vendor profits are higher with the platform, i.e., either $\pi_i^P \geq \pi_i^N$ or $\pi_i^D \geq \pi_i^N$ for all $i \in \{1, \ldots, n\}$.

Proposition 7 provides sufficient conditions under which the introduction of the platform leads to either intensified or alleviated competition among vendors. Specifically, it identifies two scenarios: Case (i), the vendor with the smallest scale (i.e., vendor 1) prefers to build a dedicated delivery fleet in the absence of per-order delivery subsidy, which results in subsidized per-order delivery services with the introduction of the platform, thereby intensifying vendor competition and increasing individual customer surplus; and Case (ii), the vendor with the largest scale (i.e., vendor n) prefers to participate on the platform in the absence of per-order delivery profiting, which leads to profiting from per order delivery service in equilibrium, thus alleviating vendor competition, benefiting vendors, and decreasing individual customer surplus. It is worth noticing that in the presence of asymmetric vendor scales, small-scale vendors can benefit from the introduction of the platform even if it intensifies vendor competition (following Definition 1, we define the introduction of the platform as intensifying vendor competition if every vendor adopts a strategy results in a lower overall price paid by its customers). This is because, due to economies of scale, the average delivery cost by building a dedicated delivery fleet is higher for smaller-scale vendors. Then, in an equilibrium with m^* vendors participating on the platform, to attract the m^*th vendor, the delivery fee charged by the platform is lower than what would be needed to attract smaller scale vendors (i.e., vendors $\{1, \ldots, m^* - 1\}$). Consequently, this further reduces delivery fees for smaller-scale vendors, which results in overall benefits for them and offsets the negative impact of intensified competition. Proposition 7 underscores the nuances and layers of our key findings in the presence of asymmetric vendor scales.

5.5. System Under Piecemeal Take-Home-Pay Per Order

In our base model, we consider a scenario where couriers receive an hourly wage. Although this setup is reasonable for many settings, as discussed in Section 3.1, there is an alternative model in which couriers receive a piecemeal take-home-pay per order without compensation for idle time while waiting for dispatch. In this section, we analyze this alternative system. Specifically, we consider a setting where couriers receive a piecemeal pay $\hat{h}$ per order. In this context, vendor i solves Problem (P) when participating on the platform or solves the following problem by establishing their own dedicated delivery fleet:

$$\max_{p_i} \quad \lambda_i(p_i - \hat{h}). \tag{15}$$

Let $\hat{\Omega}^*(p_d)$ denote the set of second-stage equilibrium profiles of vendors, given the delivery fee p_d charged to customers by the platform (note that $\hat{\Omega}^*(p_d)$ is analogous to the definition of $\Omega^*(p_d)$ in (5)). The platform sets the delivery fee p_d to maximize its profit:

$$\begin{aligned} \max_{p_d} \quad & \sum_{i=1}^{n} \lambda_i(\gamma p_i^F + p_d - \hat{h}) \cdot J_i \\ \text{subject to} \quad & p_d \in \{p_d' \mid \hat{\Omega}^*(p_d') \neq \emptyset\}, \\ & \mathcal{P} \in \hat{\Omega}^*(p_d), \\ & m = \sum_{i=1}^{n} J_i \in \{1, \ldots, n\}. \end{aligned}$$

Under the piecemeal pay scheme, the marginal cost of delivering each unit of demand is $\hat{h}$. Therefore, we say that the platform subsidizes the per-order delivery service if $p_d < \hat{h}$, and it profits per order from the delivery service otherwise. The following proposition provides a counterpart to Propositions 1 and 3 from our base model.

Proposition 8 (Piecemeal Pay)**.** *When couriers receive a piecemeal take-home-pay per order, the platform subsidizes the delivery service per order in equilibrium, i.e.,* $p_d^* < \hat{h}$*, and all vendors participate on the platform, i.e.,* $m^* = n$*. The introduction of the platform intensifies vendor competition, increasing individual customer surplus, i.e.,* $u^P \succ u^N$*, and leading to lower vendor profits, i.e.,* $\pi^P < \pi^N$*.*

Proposition 8 indicates that when couriers receive a piecemeal take-home pay per order, the introduction of the platform always intensifies vendor competition, thereby negatively affecting vendors. Unlike the setup of our base model in which couriers are paid hourly, vendors, in this case, incur only operational costs and not fixed costs. Consequently, the advantage of eliminating fixed costs by participating on the platform does not apply. Therefore, to attract vendor participation, the platform must subsidize the per-order delivery service. This subsidy reduces the vendors' operational costs, encouraging them to engage in more intense competition (by setting lower prices, i.e., $p^P < p^N$, for more demand, i.e, $\lambda^P > \lambda^N$), which in turn lowers their profits (i.e., $\pi^P < \pi^N$), as previously discussed.

6. Concluding Remarks

The growing demand for on-demand delivery services (such as food and grocery delivery) has stimulated the emergence and growth of platforms offering such services. These platforms act as aggregators, consolidating both demand and supply. Nevertheless, the relationship between the platform and vendors is fraught with tension. In this work, we construct a game-theoretical model to study how the introduction of the platform reshapes vendor competition and its subsequent impact on vendors, customers, and the platform. We investigate a system in which each vendor strategically determines whether to offer delivery services and, if so, whether to deploy in-house delivery or participate on the platform to meet the delivery window constraint. By building a dedicated delivery fleet, a vendor sets both the food price and the delivery fee to charge customers. By participating on the platform, a vendor sets the food price while the platform decides on the delivery fee, and the vendor shares a fraction of its revenue with the platform as a commission. We study the game analytically for systems with and without the platform and compare the equilibrium outcomes.

We uncover the nuanced impacts of introducing the platform. In particular, under conditions in which building dedicated delivery fleets is more favorable, such as the market potential or the maximum allowable delivery time being high, or the marginal delivery cost or the level of substitutability of vendors being low, the introduction of the platform intensifies vendor competition through per-order delivery subsidy, leading to lower vendor profits and higher individual customer surplus. Conversely, under conditions in which participating on the platform is more favorable, such as the market potential or the maximum allowable delivery time being low, or the marginal delivery cost or the level of substitutability of vendors being high, the introduction of the platform alleviates vendor competition by profiting per order from the delivery service, resulting in higher vendor profits and lower individual customer surplus.

The findings of our paper have several implications. First, we show that the seemingly appealing per-order delivery subsidy (offered by the platform to attract vendor participation) hurts vendors by compelling them to engage in more intensive competition, ultimately eroding their profits. This could potentially offer an explanation for the financial challenges and tensions that exist between the platform and vendors. The platform-enabled sharing of couriers can improve delivery efficiency and lower delivery costs but may intensify vendor competition through per-order delivery subsidy, placing them in a prisoner's dilemma that ultimately lowers their profits. Second, whether vendors benefit or are harmed by the platform entry hinges on the market competitive environment in which the vendors operate. We demonstrate that the per-order delivery subsidy is more likely to occur in areas where customers are tolerant of longer delays (e.g., Phoenix, as indicated in Table 1 reflected by the long delivery range), or the labor costs are low (e.g, Toronto, as shown in Table 1) or the level of substitutability is low (e.g., Toronto as well, which has the most diverse food scene in the world[12]). Conversely, per-order delivery profiting is more likely to occur in areas with delay-sensitive customers, high labor costs (e.g., Philadelphia, as shown in Table 1), or a high level of substitutability (e.g., Los Angeles, which is home to the most Mexican restaurants in the U.S.[13]). Such information might be helpful for social planers aiming to regulate the market to steer the outcomes resulting from the platform entry towards the desired direction, Âwhether that is favoring vendors at the expense of customers or vice versa. Lastly, the results and insights obtained from this paper have broader applicability beyond the realm of on-demand delivery businesses, such as e-commerce. For example, as discussed in Section 2, the system we examine in this paper shares

[12] https://www.restobiz.ca/toronto-food-scene-ranked-as-worlds-most-diverse/

[13] https://www.cbsnews.com/losangeles/news/la-county-home-to-the-most-mexican-restaurants-in-the-country-new-study-says/

many similarities with the "Fulfillment by Platform" program such as "Fulfillment by Amazon." This includes similar interactions between the platform (resp., Amazon) and competing vendors (resp., third-party sellers), as well as the advantage of economies of scale leveraged by the platform (resp., Amazon). Despite the distinctions between these two systems (perhaps the most salient one is that Amazon not only provides the delivery service to third-party sellers but also competes with them in the same business while sharing the benefits of gained economies of scale), our findings may offer insights into the sustainable operations of such kind of Fulfillment by Platform business.

Our model has certain limitations. First, although our model applies to systems in which the walk-in and delivery are two independent channels, we do not take into account the interactions between these two channels. Second, we consider the delivery window to be exogenously determined by the market standards and common practices, while it could also be endogenously affected by the market dynamics in a complex manner, e.g., firms compete in setting the delivery time promises. Finally, we do not account for the potential discrepancies in service quality between vendors building dedicated delivery fleets and those participating on the platform. However, one can imagine that vendors employing their own dedicated delivery fleets may offer higher service quality due to greater control over the end-to-end delivery process. Incorporating some of these ideas can be fruitful directions. We leave them for future research.

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Online Appendix to "Platform Entry and Vendor Competition in On-Demand Economy": Supplementary Derivations and Proofs

The Appendices are organized as follows. In Appendix A, we conduct the equilibrium analysis. In Appendix B, we compare systems with and without the platform. In Appendix C, we provide proofs for various extensions discussed in Section 5.

A. Equilibrium Analysis

In this section, we conduct the equilibrium analysis. We characterize the best response of vendor i given $\mathcal{P}_{-i}$ in Section A.1; we characterize the set of second-stage equilibrium profiles $\Omega^*(p_d)$ under the platform's strategy p_d in Section A.2; and we solve for the equilibrium (i.e., the Platform Problem) in Section A.3.

A.1. Proof of Lemma 1

The proof consists of 3 steps. In step (1), we characterize the highest profit that vendor i can achieve by building a dedicated delivery fleet; in step (2), we characterize that by participating on the platform; and in step (3), we show that vendor i is (weakly) better off by participating on the platform if and only if $p_d \le d(M(\mathcal{P}_{-i}))$, where $M(\mathcal{P}_{-i})$ is defined in (2) and $d(M)$ is defined in (3).

Step (1). By building a dedicated delivery fleet, vendor i solves Problem (D). We consider a modified version of the problem (in which we relax the non-negative constraint on λ_i) as follows:

$$\max_{p_i} \quad \lambda_i p_i - h\left(\frac{1}{\overline{w}} + \lambda_i\right), \tag{D'}$$

$$\text{subject to} \quad \lambda_i = M(\mathcal{P}_{-i}) - p_i. \tag{A.1}$$

Observe that if the optimal solution $\tilde{p}_i^D$ to Problem (D') satisfies $\tilde{\lambda}_i^D = M(\mathcal{P}_{-i}) - \tilde{p}_i^D \ge 0$ and $\tilde{\lambda}_i^D \tilde{p}_i^D - h\left(\frac{1}{\overline{w}} + \tilde{\lambda}_i^D\right) \ge 0$, it also solves the original Problem (D). Otherwise, $\lambda_i^D = 0$. The first order condition of (D') is given by $\frac{\partial \lambda_i}{\partial p_i}(p_i - h) + \lambda_i = -(p_i - h) + \lambda_i = 0$, which implies that

$$\lambda_i^D = p_i^D - h. \tag{A.2}$$

Combined with (A.1), we can obtain p_i^D defined in (4) in Lemma 1. By (A.2), $\lambda_i^D = p_i^D - h \ge 0$ is equivalent to $M(\mathcal{P}_{-i}) \ge h$, and $\lambda_i^D p_i^D - h\left(\frac{1}{\overline{w}} + \lambda_i^D\right) \ge 0$ is equivalent to $\overline{w} \ge \frac{4h}{[M(\mathcal{P}_{-i}) - h]^2}$.

Step (2). By participating on the platform, vendor i solves Problem (P). Similar to step (1), we consider a modified version of the problem as follows:

$$\max_{p_i} \quad (1-\gamma)\lambda_i(p_i - p_d), \tag{P'}$$

$$\text{subject to Eqn (A.1)}. \tag{A.3}$$

The first order condition of Problem (P') is given by $(1-\gamma)\left[\frac{\partial \lambda_i}{\partial p_i}(p_i - p_d) + \lambda_i\right] = (1-\gamma)[-(p_i - p_d) + \lambda_i] = 0$, which implies that

$$\lambda_i^P = p_i^P - p_d. \tag{A.4}$$

Combined with (A.1), we can obtain p_i^P defined in (4) in Lemma 1. By (A.4), $\lambda_i^P \geq 0$ is equivalent to $M(\mathcal{P}_{-i}) \geq p_d$.

Step (3). By step (1) analysis, the highest profit that vendor i can achieve by building a dedicated delivery fleet is

$$\pi_i^D = \begin{cases} \lambda_i^D(p_i^D - h) - \frac{h}{\overline{w}} = \frac{\left[M(\mathcal{P}_{-i})-h\right]^2}{4} - \frac{h}{\overline{w}} & \text{if } M(\mathcal{P}_{-i}) > h \text{ and } \overline{w} > \frac{4h}{[M(\mathcal{P}_{-i})-h]^2}, \\ 0 & \text{otherwise }. \end{cases}$$

where the last equality for the first case (i.e., $M(\mathcal{P}_{-i}) > h$ and $\overline{w} > \frac{4h}{[M(\mathcal{P}_{-i})-h]^2}$) follows from (A.2). By step (2) analysis, the highest profit that vendor i can achieve by participating on the platform is

$$\pi_i^P = \begin{cases} (1-\gamma)\lambda_i^P(p_i^P - p_d) = \frac{(1-\gamma)\left[M(\mathcal{P}_{-i})-p_d\right]^2}{4} & \text{if } M(\mathcal{P}_{-i}) > p_d, \\ 0 & \text{otherwise }. \end{cases}$$

where the last equality for first case (i.e., $M(\mathcal{P}_{-i}) > p_d$) follows from (A.4). Vendor i chooses to participate on the platform if and only if $\pi_i^P \geq \pi_i^D$. If that $M(\mathcal{P}_{-i}) > h$ and $\overline{w} > \frac{4h}{[M(\mathcal{P}_{-i})-h]^2}$, $\pi_i^P \geq \pi_i^D$ is equivalent to $p_d \leq M(\mathcal{P}_{-i}) - \left(\frac{[M(\mathcal{P}_{-i})-h]^2 - \frac{4h}{\overline{w}}}{1-\gamma}\right)^{\frac{1}{2}} = d(M(\mathcal{P}_{-i}))$, where $d(M)$ is defined in (3) and the equality holds if $p_d = d(M(\mathcal{P}_{-i}))$. Otherwise, $\pi^P \geq \pi^D = 0$, where the equality holds if $M(\mathcal{P}_{-i}) \leq p_d$.

A.2. Second-Stage Equilibrium

In this section, we characterize the set of second-stage equilibrium profiles $\Omega^*(p_d)$, which is defined in (5). For convenience, we use

$$\mathcal{P}^*(m, p_d) = (p^P(m, p_d), p^D(m, p_d), m) \tag{A.5}$$

to denote a reduced form of vendor strategy profile $\mathcal{P}$ under which m vendors participate on the platform with a (full) price $p^P(m, p_d)$, and $n-m$ vendors build dedicated delivery fleets with a (full) price $p^D(m, p_d)$, where $p^P(m, p_d)$ and $p^D(m, p_d)$ are defined in (6) and (7) respectively. We then define two series of functions $\underline{M}_m(p_d)$ for $m \in \{0, \ldots, n-1\}$ and $\overline{M}_m(p_d)$ for $m \in \{1, \ldots, n\}$ as follows

$$\underline{M}_m(p_d) = \theta + \beta\left[mp^P(m, p_d) + (n-m-1)p^D(m, p_d)\right]$$

$$= \frac{2\theta + \beta\left(\frac{2mp_d + [2(n-m-1)+\beta(n-1)]h}{2+\beta}\right)}{2-\beta(n-1)} \text{ for } m \in \{0,\ldots,n-1\}, \text{ and} \tag{A.6}$$

$$\overline{M}_m(p_d) = \theta + \beta\left[(m-1)p^P(m,p_d) + (n-m)p^D(m,p_d)\right]$$

$$= \frac{2\theta + \beta\left(\frac{[2(m-1)+\beta(n-1)]p_d + 2(n-m)h}{2+\beta}\right)}{2-\beta(n-1)} \text{ for } m \in \{1,\ldots,n\}. \tag{A.7}$$

We characterize the set of second-stage equilibrium profiles per Lemma A.1 below. Recall we define function $d(M)$ in (3) and $\mathcal{P}^*(m,p_d)$ in (A.5).

Lemma A.1. *For $m \in \{0,\ldots,n-1\}$, $d(\underline{M}_m(p_d)) = p_d$ admits a unique solution $\underline{d}_m$ with respect to p_d; and for $m \in \{1,\ldots,n\}$,$d(\overline{M}_m(p_d)) = p_d$ admits a unique solution $\overline{d}_m$ with respect to p_d. Then the set of second-stage equilibrium profiles $\Omega^*(p_d)$ is characterized as follows:*

(i) If the Subsidy Condition *is satisfied, we have $\underline{d}_0 < \cdots < \overline{d}_{m-1} < \underline{d}_{m-1} < \overline{d}_m < \underline{d}_m < \cdots < \overline{d}_n < h$, then*

- *(i.i) $\Omega^*(p_d) = \{\mathcal{P}^*(n,p_d)\}$ if $p_d < \underline{d}_0$;*
- *(i.ii) $\Omega^*(p_d) = \{\mathcal{P}^*(n,p_d), \mathcal{P}^*(0,p_d)\}$ if $p_d \in [\underline{d}_0, \overline{d}_n]$; and*
- *(i.iii) $\Omega^*(p_d) = \{\mathcal{P}^*(0,p_d)\}$ if $p_d > \overline{d}_n$.*

(ii) Otherwise, we have $h \le \overline{d}_n \le \cdots \le \underline{d}_m \le \overline{d}_m \le \underline{d}_{m-1} \le \overline{d}_{m-1} \le \cdots \le \underline{d}_0$, where the equalities hold if $\frac{h}{w} = \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, then

- *(ii.i) $\Omega^*(p_d) = \{\mathcal{P}^*(n,p_d)\}$ if $p_d \le \overline{d}_n$;*
- *(ii.ii) $\Omega^*(p_d) = \{\mathcal{P}^*(m,p_d)\}$ if $p_d \in [\underline{d}_m, \overline{d}_m]$ for $m \in \{1,\ldots n-1\}$;*
- *(ii.iii) $\Omega^*(p_d) = \{\mathcal{P}^*(0,p_d)\}$ if $p_d \ge \underline{d}_0$; and*
- *(ii.iv) $\Omega^*(p_d) = \emptyset$ otherwise.*

Discussion on the refinement rule. By Lemma A.1, when the Subsidy Condition is satisfied, a delivery fee $p_d \in [\underline{d}_0, \overline{d}_n]$ induces two second-stage equilibrium profiles: one in which no vendors participate on the platform, i.e., $\mathcal{P}^*(0,p_d)$, and one in which all vendors participate on the platform, i.e., $\mathcal{P}^*(n,p_d)$. Based on the our refinement rule introduced in Section 3.2, we select $\mathcal{P}^*(n,p_d)$. When $\frac{h}{w} = \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, $p_d = h$ induces n second-stage equilibrium profiles (in which the platform has a positive market share) with varying numbers of vendors participating on the platform, i.e., $\mathcal{P}^*(m,h)$ for $m \in \{1,\ldots,n\}$. In this case, we have $\Pi(m,h) = m\gamma\lambda(m,h)(p^P(m,h) - h) - \frac{h}{w} = n\gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2 - \frac{h}{w} = (m-1)\gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, which increases in m. Based on our refinement rule, we select $\mathcal{P}^*(n,h)$. Therefore, the refinement rule aligns with selecting the second-stage equilibrium with the largest number of vendors participating on the platform.

In the remainder of this section, we provide the Proof of Lemma A.1.

Proof of Lemma A.1. We first define three terminologies. We say that a strategy profile $\mathcal{P}$ is *group-wise symmetric* if vendors with the same platform participation decisions play the same strategy (i.e., vendors who participate on the platform charge the same food price, and those who build dedicated delivery fleets charge the same full price). Moreover, we refer to a deviation under which vendor i only changes its pricing strategy but maintains its platform participation decision as a *within-group deviation*, and that under which vendor i changes its platform participation decision as a *cross-group deviation*.

We then prove Lemma A.1 by 3 steps. In step (1), we show that a second-stage equilibrium profile must be group-wise symmetric. In step (2), we characterize conditions that ensure neither within-group deviations nor cross-group deviations. In step (3), we analyze the conditions obtained from step (2) in detail and derive $\Omega^*(p_d)$.

Step (1). Suppose (for contradiction) there exists a second-stage equilibrium profile $\mathcal{P}$ with both vendor i and j building dedicated delivery fleets while $p_i \neq p_j$. By (4), we must have

$$p_i = \frac{\theta + \beta p_j + \beta \sum_{l \neq i,j} p_l + h}{2} \text{ and } p_j = \frac{\theta + \beta p_i + \beta \sum_{l \neq i,j} p_l + h}{2}.$$

Observe that the above system of equations is infeasible if $p_i \neq p_j$. Therefore, under a second-stage equilibrium profile, vendors building dedicated delivery fleets must charge customers the same full price. Similarly, one can show that the full prices associated with vendors who participate on the platform must be the same. Then, it suffices to focus on group-wise symmetric strategy profiles. We can then describe the strategy profile of vendors with the reduced form $\mathcal{P} = (p^P, p^D, m)$, where p^P and p^D are the (full) prices associated with vendors participating on the platform and building dedicated delivery fleets respectively and $m \in \{0, \dots, n\}$ is the number of vendors participating on the platform. We also describe the strategy profile of vendors except for vendor i similarly with $\mathcal{P}_{-i} = (p^P, p^D, m')$, where $m' \in \{0, \dots, n-1\}$ is the number of vendors participating on the platform without considering vendor i. Then observe from (2) and (A.6)–(A.7), given $\mathcal{P}_{-i} = (p^P(m, p_d), p^D(m, p_d), m)$, the competitive market potential of vendor i is $\underline{M}_m(p_d) = M(\mathcal{P}_{-i})$, and given $\mathcal{P}_{-i} = (\mathcal{P}^P(m, p_d), p^D(m, p_d), m-1)$, that of vendor i is $\overline{M}_m(p_d) = M(\mathcal{P}_{-i})$.

Step (2). We consider the within-group deviation and the cross-group deviation separately.

Within-Group Deviation. Recall from Lemma 1 the set of best response strategies of vendor i given $\mathcal{P}_{-i}$. Also recall that we define two series of functions $\underline{M}_m(p_d)$ for $m \in \{0, \dots, n-1\}$ and $\overline{M}_m(p_d)$ for $m \in \{1, \dots, n\}$ in (A.6)–(A.7) respectively. Provided that there are m vendors participating on the platform, by (4), the conditions that ensure no within-group deviations under $\mathcal{P} = (p^P, p^D, m)$ are as follows:

$$p^P = \frac{\overline{M}_m(p_d) + p_d}{2} = \frac{\theta + \beta\left[(m-1)p^P + (n-m)p^D\right] + p_d}{2} \quad \text{and}$$

$$p^D = \frac{\underline{M}_m(p_d) + h}{2} = \frac{\theta + \beta\left[mp^P + (n-m-1)p^D\right] + h}{2}.$$

The reason is as follows. If vendor i chooses to participate on the platform, it sees $\mathcal{P}_{-i} = (p^P, p^D, m-1)$. Then its strategy p^P must satisfy the optimality condition of Problem (P'). By the expression of p_i^P in (4) and the definition of $\overline{M}_m(p_d)$ in (A.7), we can obtain the first condition. Similarly, if vendor i chooses to build a dedicated delivery fleet, it sees $\mathcal{P}_{-i} = (p^P, p^D, m)$. Then its strategy p^D must satisfy the optimality condition of Problem (D'). By the expression of p_i^D in (4) and the definition of $\underline{M}_m(p_d)$ in (A.6), we can obtain the second condition. We use $p^P(m, p_d)$ and $p^D(m, p_d)$ to indicate the dependence of the solution to the system of equations on m and p_d. We can then obtain (6)–(7).

Cross-Group Deviation. By the no within-group deviation analysis, it suffices to focus on the vendor strategy profiles $\mathcal{P}^*(m, p_d)$ as specified in (A.5). By Lemma 1, vendor i chooses to participate on the platform if $p_d \le d(M(\mathcal{P}_{-i}))$, and it chooses to build a dedicated delivery fleet if $p_d \ge d(M(\mathcal{P}_{-i}))$, where $M(\mathcal{P}_{-i})$ and $d(M)$ are defined in (2) and (3) respectively. Given that the system has m vendors participating on the platform, the condition that ensures no cross-group deviations is given as follows

$$\begin{cases} p_d \ge d(\underline{M}_0(p_d)) & \text{if } m = 0, \\ d(\underline{M}_m(p_d)) \le p_d \le d(\overline{M}_m(p_d)) & \text{if } m \in \{1, \ldots, n-1\}, \\ p_d \le d(\overline{M}_n(p_d)) & \text{if } m = n, \end{cases} \tag{A.8}$$

where $\underline{M}_m(p_d)$ for $m \in \{0, \ldots, n-1\}$ and $\overline{M}_m(p_d)$ for $m \in \{1, \ldots, n\}$ are defined respectively in (A.6)–(A.7). The reason is as follows. First, we need to ensure that a vendor, say vendor i, who participates on the platform, has no incentive to exit and build a dedicated delivery fleet. As vendor i sees $\mathcal{P}_{-i} = (p^P(m, p_d), p^D(m, p_d), m-1)$, this condition is given by $p_d \le d(\overline{M}_m(p_d))$ by Lemma 1 and the definition of $\overline{M}_m(p_d)$. Similarly, we need to ensure that vendor i, who builds a dedicated delivery fleet, has no incentive to deviate and participate on the platform. As vendor i sees $\mathcal{P}_{-i} = (p^P(m, p_d), p^D(m, p_d), m)$, this condition is given by $p_d \ge d(\underline{M}_m(p_d))$ by Lemma 1 and the definition of $\underline{M}_m(p_d)$.

We then introduce some preliminary results per Lemma A.2 below.

Lemma A.2. *We have the following results:*

(a) $d(M)$ *is decreasing, where* $d(M)$ *is defined in* (3)*;*

(b) both $d(\underline{M}_m(p_d)) - p_d$ *for* $m \in \{0, \ldots, n-1\}$ *and* $d(\overline{M}_m(p_d)) - p_d$ *for* $m \in \{1, \ldots n\}$ *are decreasing in* p_d*, where* $\underline{M}_m(p_d)$ *and* $\overline{M}_m(p_d)$ *are defined in* (A.6)–(A.7)*; and*

(c) depending on the value of p_d*, we have*

(c.i) $\underline{M}_0(h) = \cdots = \underline{M}_m(h) = \overline{M}_m(h) = \cdots = \overline{M}_n(h) = \frac{\beta(n-1)(\theta+h)}{2-\beta(n-1)}$ *if* $p_d = h$;

(c.ii) $\overline{M}_n(p_d) < \cdots < \underline{M}_m(p_d) < \overline{M}_m(p_d) < \underline{M}_{m-1}(p_d) < \overline{M}_{m-1}(p_d) < \cdots < \underline{M}_0(p_d)$ *if* $p_d < h$*; and*

(c.iii) $\overline{M}_n(p_d) > \cdots > \underline{M}_m(p_d) > \overline{M}_m(p_d) > \underline{M}_{m-1}(p_d) > \overline{M}_{m-1}(p_d) > \cdots > \underline{M}_0(p_d)$ *if* $p_d > h$.

Proof of Lemma A.2. We prove each result separately.

(a) The result follows from $d'(M) = 1 - \left(\frac{M-h}{1-\gamma}\right) \Big/ \sqrt{\frac{[M-h]^2 - \frac{4h}{w}}{1-\gamma}} < 0$.

(b) The result follows from the result in (a) and the fact that both $\underline{M}_m(p_d)$ and $\overline{M}_m(p_d)$ are increasing in p_d by (A.6)–(A.7).

(c) By (A.6) – (A.7), we can obtain that $\underline{M}_m(p_d) - \overline{M}_m(p_d) = \frac{\beta(p_d-h)}{2+\beta}$ and $\overline{M}_m(p_d) - \underline{M}_{m-1}(p_d) = \frac{\beta^2(n-1)(p_d-h)}{(2+\beta)[2-\beta(n-1)]}$. Then, the results follow directly. □

Recall we define $\underline{d}_m$ as the unique solution to $d(\underline{M}_m(p_d)) = p_d$ for $m \in \{0, \ldots, n-1\}$ and $\overline{d}_m$ as the unique solution to $d(\overline{M}_m(p_d)) = p_d$ for $m \in \{1, \ldots, n\}$ in Lemma A.1 (the proofs for the uniqueness of $\underline{d}_m$ and $\overline{d}_m$ are provided in Step (3) analysis). By (b) of Lemma A.2, (A.8) is equivalent to

$$\begin{cases} p_d \geq \underline{d}_0 & \text{if } m = 0, \\ \underline{d}_m \leq p_d \leq \overline{d}_m & \text{if } m \in \{1, \ldots, n-1\}, \\ p_d \leq \overline{d}_n & \text{if } m = n. \end{cases} \tag{A.9}$$

Step (3). We consider the following scenarios.

Case (i) The Subsidy Condition is satisfied. Define $\underline{d}_m^{min} = \frac{2+\beta}{2m\beta}\left([2-\beta(n-1)]\left(2\sqrt{\frac{h}{w}}+h\right) - 2\theta\right) - \frac{[2(n-m-1)+\beta(n-1)]h}{2m}$ and $\overline{d}_m^{min} = \frac{2+\beta}{\beta[2(m-1)+\beta(n-1)]}\left([2-\beta(n-1)]\left(2\sqrt{\frac{h}{w}}+h\right) - 2\theta\right) - \frac{2(n-m)h}{2(m-1)+\beta(n-1)}$. Observe that $\underline{d}_m^{min}$ is the unique solution to $\left(\underline{M}_m(p_d) - h\right)^2 - \frac{4h}{w} = 0$ and $\overline{d}_m^{min}$ is the unique solution to $\left(\overline{M}_m(p_d) - h\right)^2 - \frac{4h}{w} = 0$. Therefore, $\underline{d}_m^{min}$ and $\overline{d}_m^{min}$ can be interpreted as the minimum delivery fee (can be negative) charged by the platform so that $d(\underline{M}_m(p_d))$ and $d(\overline{M}_m(p_d))$ are well defined, where $d(M)$ is defined in (3).

We first note that

$$d(\underline{M}_m(\underline{d}_m^{min})) - \underline{d}_m^{min} > 0 \text{ and } d(\overline{M}_m(\overline{d}_m^{min})) - \overline{d}_m^{min} > 0. \tag{A.10}$$

The reason is as follows. By the Vendor Viability Condition, we have$d(\underline{M}_m(\underline{d}_m^{min})) - \underline{d}_m^{min} = \left(2\sqrt{\frac{h}{w}}+h\right) + \frac{2+\beta}{2m\beta}\left(2\theta - [2-\beta(n-1)]\left(2\sqrt{\frac{h}{w}}+h\right)\right) + \frac{[2(n-m-1)+\beta(n-1)]h}{2m} > 0$ and $d(\overline{M}_m(\overline{d}_m^{min})) - \overline{d}_m^{min} = \left(2\sqrt{\frac{h}{w}}+h\right) + \frac{2+\beta}{\beta[2(m-1)+\beta(n-1)]}\left(2\theta - [2-\beta(n-1)]\left(2\sqrt{\frac{h}{w}}+h\right)\right) + \frac{2(n-m)h}{2(m-1)+\beta(n-1)} > 0$.

Then we can obtain that $d(\overline{M}_n(h)) - h = d(\frac{\beta(n-1)(\theta+h)}{2-\beta(n-1)}) - h = \frac{2(\theta-[1-\beta(n-1)]h)}{2-\beta(n-1)} - \sqrt{\frac{\left(\frac{2(\theta-[1-\beta(n-1)]h)}{2-\beta(n-1)}\right)^2 - \frac{4h}{w}}{1-\gamma}} < 0$, where the inequality follows from the Subsidy Condition. Combined with the facts that $d(\overline{M}_n(\overline{d}_n^{min}) - \overline{d}_n^{min} > 0$ by (A.10) and $d(\overline{M}_n(p_d)) - p_d$ decreasing in p_d by (b) of Lemma A.2, $d(\overline{M}_n(p_d)) - p_d = 0$ admits a unique root $\overline{d}_n \in (\overline{d}_n^{min}, h)$ by the Intermediate value theorem. By Lemma A.2, we have $d(\underline{M}_{n-1}(\overline{d}_n)) < d(\overline{M}_n(\overline{d}_n)) = \overline{d}_n$. Then by (b) of Lemma A.2, $d(\underline{M}_{n-1}(p_d)) = p_d$ admits a unique solution $\underline{d}_{n-1} \in (\underline{d}_{n-1}^{min}, \overline{d}_n)$. Similarly, we have $d(\overline{M}_{n-1}(\underline{d}_{n-1})) < d(\underline{M}_{n-1}(\underline{d}_{n-1})) = \underline{d}_{n-1}$, and thus $d(\overline{M}_{n-1}(p_d)) = p_d$ admits a unique solution $\overline{d}_{n-1} \in (\overline{d}_{n-1}^{min}, \underline{d}_{n-1})$. By repeating this analysis recursively, we can establish the uniqueness of $\underline{d}_m$ for $m \in \{0, \dots, n-1\}$ and $\overline{d}_m$ for $m \in \{1, \dots, n\}$, and $\underline{d}_0 < \dots < \overline{d}_{m-1} < \underline{d}_{m-1} < \overline{d}_m < \underline{d}_m < \dots < \overline{d}_n < h$.

Because $\underline{d}_m > \overline{d}_m$, there does not exist a p_d satisfying (A.9) for $m \in \{1, \dots, n-1\}$. Therefore, $\Omega^*(p_d) = \{\mathcal{P}^*(n, p_d)\}$ if $p_d < \underline{d}_0$, $\Omega^*(p_d) = \{\mathcal{P}^*(0, p_d)\}$ if $p_d > \overline{d}_n$ and $\Omega^*(p_d) = \{\mathcal{P}^*(0, p_d), \mathcal{P}^*(n, p_d)\}$ if $p_d \in [\underline{d}_0, \overline{d}_n]$.

Case (ii) The Subsidy Condition is not satisfied. Let $\lambda^P(m, p_d)$ denote the demand associated with vendors participating on the platform under $\mathcal{P}^*(m, p_d)$. For convenience, we use $p_d^{max}(m)$ to denote the solution to $\lambda^P(m, p_d) = p^P(m, p_d) - p_d = \frac{\overline{M}_m(p_d) - p_d}{2} = 0$, where the first equation follows from (A.4) and the second equation follows from (4). Therefore, $p_d^{max}(m)$ can be interpreted as the highest delivery fee the platform can charge such that vendors participating on the platform have positive market shares. We then show that $d(\overline{M}_n(p_d)) = p_d$ admits a unique solution $\overline{d}_n \in [h, p_d^{max}(n))$. By (b) of Lemma A.2, this is equivalent to show that $d(\overline{M}_n(h)) \geq h$ and $d(\overline{M}_n(p_d^{max}(n))) < p_d^{max}(n)$. The first inequality follows as the Subsidy Condition is not satisfied. We then show that the second inequality holds:

$$
\begin{aligned}
d(\overline{M}_n(p_d^{max}(n))) &= \overline{M}_n(p_d^{max}(n)) - \left(\frac{[\overline{M}_n(p_d^{max}(n)) - h]^2 - \frac{4h}{\overline{w}}}{1-\gamma} \right)^{\frac{1}{2}} \\
&= p_d^{max}(n) - \left(\frac{[\overline{M}_n(p_d^{max}(n)) - h]^2 - \frac{4h}{\overline{w}}}{1-\gamma} \right)^{\frac{1}{2}} \\
&< p_d^{max}(n),
\end{aligned}
$$

where the second equality follows from the definition of $p_d^{max}(m)$, i.e., $p_d^{max}(n)$ solves $\overline{M}_n(p_d) - p_d = 0$.

We then show that $(\underline{M}_{n-1}(p_d)) = p_d$ admits a unique solution $\underline{d}_{n-1} \in [\overline{d}_n, p_d^{max}(n-1))$, which is equivalent to show $d(\underline{M}_{n-1}(\overline{d}_n)) \geq \overline{d}_n$ and $d(\underline{M}_{n-1}(p_d^{max}(n-1))) < p_d^{max}(n-1)$ by (b) of Lemma A.2. We first prove the first inequality. By Lemma A.2 and the fact that $\overline{d}_n \geq h$, we can obtain that $d(\underline{M}_{n-1}(\overline{d}_n)) > d(\overline{M}_n(\overline{d}_n)) = \overline{d}_n$. We then prove the second inequality:

$$
\begin{aligned}
d(\underline{M}_{n-1}(p_d^{max}(n-1))) &< d(\overline{M}_{n-1}(p_d^{max}(n-1))) \\
&= \overline{M}_{n-1}(p_d^{max}(n-1)) - \left(\frac{[\overline{M}_{n-1}(p_d^{max}(n-1)) - h]^2 - \frac{4h}{\overline{w}}}{1-\gamma} \right)^{\frac{1}{2}}
\end{aligned}
$$

$$= p_d^{max}(n-1) - \left(\frac{[\overline{M}_{n-1}(p_d^{max}(n-1)) - h]^2 - \frac{4h}{\overline{w}}}{1-\gamma} \right)^{\frac{1}{2}}$$
$$< p_d^{max}(n-1),$$

where the first inequality follows from Lemma A.2 and the fact that $p_d^{max}(n-1) > h$.

Similarly, as $d(\overline{M}_{n-1}(p_d^{max}(n-1))) < p_d^{max}(n-1)$ by the above analysis, and $d(\overline{M}_{n-1}(\underline{d}_{n-1})) \geq d(\underline{M}_{n-1}(\underline{d}_{n-1})) = \underline{d}_{n-1}$ by Lemma A.2 and the fact that $\underline{d}_{n-1} > \overline{d}_n > h$, $d(\overline{M}_{n-1}(p_d)) = p_d$ admits a unique solution $\overline{d}_{n-1} \in [\underline{d}_{n-1}, p_d^{max}(n-1))$. By repeating this analysis recursively, we can establish the uniqueness of $\underline{d}_m$ for $m \in \{0, \ldots, n-1\}$ and $\overline{d}_m$ for $m \in \{1, \ldots, n\}$, and $\overline{d}_n \leq \cdots \leq \underline{d}_m \leq \overline{d}_m \leq \underline{d}_{m-1} \leq \overline{d}_{m-1} \leq \cdots < \underline{d}_0$. Moreover, By (c.iii) of Lemma A.2, these inequalities are binding only when $\frac{h}{\overline{w}} = \gamma \left(\frac{\theta - [1-\beta(n-1)]h}{2-\beta(b-1)} \right)$.

By (A.9), $\Omega^*(p_d) = \{\mathcal{P}^*(n, p_d)\}$ if $p_d \leq \overline{d}_n$, $\Omega^*(p_d) = \{\mathcal{P}^*(0, p_d)\}$ if $p_d \geq \underline{d}_0$, $\Omega^*(p_d) = \{\mathcal{P}^*(m, p_d)\}$ if $p_d \in [\underline{d}_m, \overline{d}_m]$ and $\Omega^*(p_d) = \emptyset$ otherwise, i.e., $p_d \in (\overline{d}_m, \underline{d}_{m-1})$, where $m \in \{1, \ldots, n-1\}$.

A.3. Proof of Proposition 1

As $p_i^F = p_i - p_d$, the Platform Problem can be rewritten as $\Pi(p_d) = \sum_{i=1}^{n} \lambda_i \left[\gamma p_i + (1-\gamma) p_d\right] \cdot J_i - h \cdot \mu(\sum_{i=1}^{n} \lambda_i \cdot J_i; \overline{w})$. Abusing notation, we define

$$\Pi(m, p_d) = m\lambda^P(m, p_d) \left[\gamma p^P(m, p_d) + (1-\gamma) p_d - h\right] - \frac{h}{\overline{w}}. \tag{A.11}$$

Here, recall that $\lambda^P(m, p_d)$ is the demand associated with vendors participating on the platform under vendor strategy profile $\mathcal{P}^*(m, p_d)$ defined in (A.5). By (A.4), we have $\lambda^P(m, p_d) = p^P(m, p_d) - p_d$. Then by Lemma A.1, the Platform Problem reduces to:

$$p_d^* = \arg\max_{m \in \{1, \ldots, n\}} \Pi(m, p_d^*(m)), \tag{A.12}$$

where $p_d^*(m)$ solves the following problem:

$$\max_{p_d} \quad \Pi(m, p_d),$$
$$\text{subject to} \quad \text{Eqn (A.9)}.$$

We then show that $\Pi(m, p_d)$ is concave in p_d:

$$\frac{\partial \Pi(m, p_d)}{\partial p_d} = m \left[\underbrace{\frac{\theta + \frac{\beta(n-m)(h-p_d)}{2+\beta} - [1-\beta(n-1)]p_d}{2-\beta(n-1)}}_{temp} \left(1 - \frac{2\gamma \left(\frac{\beta(n-m)}{2+\beta} + [1-\beta(n-1)] \right)}{2-\beta(n-1)} \right) \right. \tag{A.13}$$

$$-\left(\frac{\frac{\beta(n-m)}{2+\beta}+[1-\beta(n-1)]}{2-\beta(n-1)}\right)(p_d-h)\Bigg], \text{and}$$

$$\frac{\partial^2\Pi(m,p_d)}{\partial p_d^2}=-2m\left(\frac{\frac{\beta(n-m)}{2+\beta}+[1-\beta(n-1)]}{2-\beta(n-1)}\right)\left(1-\frac{\gamma\left(\frac{\beta(n-m)}{2+\beta}+[1-\beta(n-1)]\right)}{2-\beta(n-1)}\right)<0,$$

where the inequality follows from the dominant diagonal condition $\beta(n-1)<1$. Observe that $\frac{\partial\Pi(m,h)}{\partial p_d}=m\left[\frac{\theta-[1-\beta(n-1)]p_d}{2-\beta(n-1)}\left(1-\frac{2\gamma\left(\frac{\beta(n-m)}{2+\beta}+[1-\beta(n-1)]\right)}{2-\beta(n-1)}\right)\right]>0$, where the inequality follows from the Vendor Viability Condition, and $\frac{\partial\Pi(m,\hat{p}_d(m)))}{\partial p_d}=-\left(\frac{\frac{\beta(n-m)}{2+\beta}+[1-\beta(n-1)]}{2-\beta(n-1)}\right)(\hat{p}_d(m)-h)<0$, where $\hat{p}_d(m)=\frac{(2+\beta)\theta+\beta(n-m)h}{(2+\beta)[1-\beta(n-1)]+\beta(n-m)}$ uniquely solves $temp=0$ (defined in (A.13)) and $\hat{p}_d(m)>h$ by the Vendor Viability Condition. As $\frac{\partial^2\Pi(m,p_d)}{\partial p_d}<0$, $\frac{\partial\Pi(m,p_d)}{\partial p_d}=0$ admits a unique root $\tilde{p}_d(m)$ and

$$\tilde{p}_d(m)>h. \tag{A.14}$$

By Lemma A.1, when the Subsidy Condition is satisfied, $p_d\in[\underline{d}_0,\overline{p}_n]$ induces two second-stage equilibrium profiles: $\Omega^*(p_d)=\{\mathcal{P}^*(0,p_d),\mathcal{P}^*(n,p_d)\}$. By the refinement rule (under which we select the second-stage equilibrium that leads to the largest number of vendors participating on the platform, i.e., we select $\mathcal{P}^*(n,p_d)$ in this case), we can obtain that

$$p_d^*(m)=\begin{cases}\tilde{p}_d(m), & \text{if } \tilde{p}_d(m)\in(\underline{d}_m,\overline{d}_m),\\ \underline{d}_m, & \text{if } \tilde{p}_d(m)\le\underline{d}_m,\\ \overline{d}_m, & \text{if } \tilde{d}(m)\ge\overline{d}_m,\end{cases}\qquad \text{for } m\in\{1,\dots,n-1\}, \text{ and} \tag{A.15}$$

$$p_d^*(n)=\begin{cases}\tilde{p}_d(n), & \text{if } \tilde{p}_d(n)<\overline{d}_n,\\ \overline{d}_n, & \text{otherwise},\end{cases} \tag{A.16}$$

where $\underline{d}_m$ and $\overline{d}_m$ are defined in Lemma A.1. We then consider the following cases.

Case (i) The Subsidy Condition is satisfied. By Lemma A.1, it suffices to consider the case where $m=n$, and thus the platform's optimal strategy is given by $p_d^*(n)$. By (A.14) and (A.16), we have $\tilde{p}_d(n)>h$ and thus $p_d^*(n)=\overline{d}_n<h$. Therefore, by Lemma A.1, the platform subsidizes the delivery service per order, i.e., $p_d^*<h$, and all vendors participate on the platform in equilibrium, i.e., $m^*=n$.

Case (ii) The Subsidy Condition is not satisfied. We first establish the existence of an equilibrium. This result follows as the platform's optimal strategy p_d^* draws from a finite set of strategies by (A.12) and (A.15)–(A.16). We then show that $p_d^*\ge h$. For $m=n$, by Lemma A.1 and (A.14), we have $\min(\overline{d}_n,\tilde{p}_d(n))\ge h$. Then by (A.16), $p_d^*(n)\ge h$. For $m\in\{1,\dots,n-1\}$, by Lemma A.1, we have $\overline{d}_m\ge\underline{d}_m\ge h$. Then by (A.15), $p_d^*(m)\ge h$ as $p_d^*(m)\in[\underline{d}_m,\overline{d}_m]$. Because $p_d^*\in\{p_d^*(m)$ for $m\in$

$\{1,\ldots,n\}\}$, $p_d^* \geq h$. Therefore, by Lemma A.1, the platform generates profit per order from the delivery service in equilibrium, i.e., $p_d^* \geq h$. Moreover, depending on the system parameters, the equilibrium outcome can be either symmetric, i.e., $m^* = n$, or asymmetric, i.e., $m^* < n$.

B. Compare Systems With and Without The Platform

In Section B.1, we conduct the equilibrium analysis for the system without the platform. In Section B.2, we compare the equilibrium outcomes for systems with and without the platform. In Section B.3, we investigate the platform's profitability under the optimal solution to the Platform Problem.

B.1. Proof of Proposition 2

By following the analysis of no within-group deviations in the Proof of Lemma A.1, in equilibrium in which all vendors have a positive market share, i.e., $\lambda_i > 0$ for all $i \in \{1,\ldots,n\}$, vendors charges the same (full) price. Then by (7) and letting $m = 0$, we can obtain $p^N = \frac{\theta+h}{2-\beta(n-1)}$ as shown in (9). By (A.2), the corresponding demand rate is $\lambda^N = \frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}$. By (D), vendors' profit $\pi^N = \lambda^N(p^N - h) - \frac{h}{w} = \left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2 - \frac{h}{w} > 0$ if and only if the Vendor Viability Condition is satisfied.

When the Vendor Viability Condition is not satisfied, obviously, there does not exist an equilibrium in which all vendors have positive market shares. We then show that when $\theta \leq 2[1 - \beta(n-1)]\sqrt{\frac{h}{w}} + [1-\beta(n-1)]h$, there exists an equilibrium in which all vendors make 0 profits. Let all vendors except for vendor i charge a price $\overline{p} = \frac{\theta}{1-\beta(n-1)}$. Observe from (1) that $\lambda_i = 0$ for $i \in \{1,\ldots,n\}$ when all vendors charge a full price $\overline{p}$. Then, vendor i's profit is given by

$$\pi_i = \begin{cases} 0 & \text{if } p_i \geq \overline{p}, \\ \left(\theta - p_i + \frac{\beta(n-1)\theta}{1-\beta(n-1)}\right)(p_i - h) - \frac{h}{w} & \text{otherwise} . \end{cases}$$

Let $f(p_i) = \left(\theta - p_i + \frac{\beta(n-1)\theta}{1-\beta(n-1)}\right)(p_i - h) - \frac{h}{w}$. Observe that $f(p_i)$ is concave, with the maximum being achieved at $\tilde{p}_i = \frac{1}{2}\left(\frac{\theta}{1-\beta(n-1)} + h\right)$. If $\tilde{p}_i > \overline{p}$, which is equivalent to $h > \frac{\theta}{1-\beta(n-1)}$, $p_i^* = \overline{p}$ and thus vendor i has no incentive to deviate from $\overline{p}$. Otherwise, $p_i^* = \tilde{p}_i$. When $\theta \leq 2[1-\beta(n-1)]\sqrt{\frac{h}{w}} + [1+\beta(n-1)]h$, we have $\pi_i(p_i^*) = \left(\frac{\theta-[1-\beta(n-1)]h}{2[1-\beta(n-1)]}\right)^2 - \frac{h}{w} \leq 0$, and thus vendor i has no incentive to deviate from $\overline{p}$. Therefore, there exists an equilibrium in which all vendors charge a (full) price $\overline{p}$.

B.2. Proof of Proposition 3

Recall that we use p^P, π^P, λ^P (p^D, π^D, λ^D, p^N, π^N, λ^N) to denote respectively the (full) price, vendor profit and customer demand associated with vendors participating on the platform (building dedicated delivery fleets, in the system without the platform). Also, recall we provide definitions on individual customer surplus being higher or lower in Definition 1. We first compare the (full) price, vendor profit, and customer demand separately.

Compare (full) prices. We first compare the price associated with vendors building dedicated delivery fleets (in the system with the platform) with that for the system without the platform. By (7) and (9),

$$p^D - p^N = \frac{\theta + p_d^* + \frac{[2-\beta(m-1)](h-p_d^*)}{2+\beta}}{2-\beta(n-1)} - \frac{\theta + h}{2-\beta(n-1)} = \frac{\beta m(p_d^* - h)}{[2-\beta(n-1)](2+\beta)}.$$

By Proposition 1, such a comparison makes sense only if the Subsidy Condition is not satisfied (otherwise, all vendors participate on the platform in equilibrium). In this case, as $p_d^* \geq h$, we have $p^D \geq p^N$.

We then compare the price associated with vendors participating on the platform with that for the system without the platform. By (6) and (9),

$$p^P - p^N = \frac{\theta + p_d^* + \frac{\beta(n-m)(h-p_d^*)}{2+\beta}}{2-\beta(n-1)} - \frac{\theta + h}{2-\beta(n-1)} = \frac{[2-\beta(n-m-1)](p_d^* - h)}{[2-\beta(n-1)](2+\beta)}.$$

By Proposition 1, $p_d^* < h$ if the Subsidy Condition is satisfied and $p_d^* \geq h$ otherwise. Therefore, $p^P < p^N$ if the Subsidy Condition is satisfied and $p^P \geq p^N$ otherwise.

Compare vendor profits. We first compare the profit associated with vendors building dedicated delivery fleets with that for the system without the platform. By (D), (7) and (9),

$$\pi^D - \pi^N = \lambda^D \left(p^D - h\right) - \frac{h}{\overline{w}} - \left(\lambda^N \left(p^N - h\right) - \frac{h}{\overline{w}}\right) \overset{(a)}{=} \left(p^D - h\right)^2 - \left(p^N - h\right)^2,$$

where (a) follows from (A.4). By Proposition 1, such a comparison makes sense only if the Subsidy Condition is not satisfied. In this case, we have $p^D \geq p^N \geq h$, where the first inequality follows from the comparison between p^D and p^N and the second inequality follows from the Vendor Viability Condition. Therefore, $\pi^D \geq p^N$.

We then compare the profit associated with vendors participating on the platform with that for the system without the platform. By (P), (6) and (9),

$$\begin{aligned}
\pi^P - \pi^N &= (1-\gamma)\lambda^P \cdot (p^P - p_d^*) - \left(\lambda^N (p^N - h) - \frac{h}{\overline{w}}\right) \\
&\overset{(a)}{=} (1-\gamma)\left(p^P - p_d^*\right)^2 - \left(\lambda^N (p^N - h) - \frac{h}{\overline{w}}\right) \\
&\overset{(b)}{=} (1-\gamma)\left(\frac{\overline{M}_m(p_d^*) - p_d^*}{2}\right)^2 - \left(\lambda^N (p^N - h) - \frac{h}{\overline{w}}\right), \qquad \text{(B.1)}
\end{aligned}$$

where (a) follows from (A.4), and (b) follows from (4) and the definition of $\overline{M}_m(p_d)$ in (A.7). We then consider the following cases separately.

Case (i) The Subsidy Condition is satisfied. In this case, it suffices to consider the case where $m=n$ by Proposition 1. Then from (B.1), we have

$$\begin{aligned}
\pi^P-\pi^N &\overset{(c)}{=} (1-\gamma)\left(\frac{\overline{M}_n(\overline{d}_n)-\overline{d}_n}{2}\right)^2-\left(\lambda^N(p^N-h)-\frac{h}{\overline{w}}\right)\\
&\overset{(d)}{=}\left(\frac{\overline{M}_n(\overline{d}_n)-h}{2}\right)^2-\frac{h}{\overline{w}}-\left(\lambda^N(p^N-h)-\frac{h}{\overline{w}}\right)\\
&=\frac{1}{4}\left(\frac{2\theta-[2-\beta(n-1)]h+\beta(n-1)\overline{d}_n}{2-\beta(n-1)}\right)^2-\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2\\
&\overset{(e)}{<}0,
\end{aligned}$$

where (c) follows from $p_d^*=\overline{d}_n$ when the Subsidy Condition is satisfied (this result is derived from Proposition 1), (d) follows from the definition of $\overline{d}_n$ in Lemma A.1, and (e) follows from $\overline{d}_n<h$ by Lemma A.1.

Case (ii) The Subsidy Condition is not satisfied. Then from (B.1), we have

$$\begin{aligned}
\pi^P-\pi^N &\overset{(f)}{\geq} (1-\gamma)\left(\frac{\overline{M}_m(\overline{d}_m)-\overline{d}_m}{2}\right)^2-\left(\lambda^N(p^N-h)-\frac{h}{\overline{w}}\right)\\
&\overset{(g)}{=}\left(\frac{\overline{M}_m(\overline{d}_m)-h}{2}\right)^2-\frac{h}{\overline{w}}-\left(\lambda^D(p^D-h)-\frac{h}{\overline{w}}\right)\\
&=\frac{1}{4}\left(\frac{2\theta+\frac{\beta[2(m-1)+\beta(n-1)]\overline{d}_m+2\beta(n-m)h}{2+\beta}}{2-\beta(n-1)}-h\right)^2-\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2\\
&\geq 0
\end{aligned}$$

if $\overline{d}_m>h$. Here, (f) follows from $p_d^*\in\{p_d^*(m)\text{ for } m\in\{1,\dots,n\}\}\leq\overline{d}_m$ by (A.15) –(A.16) and $\overline{M}_m(p_d)-p_d$ being decreasing in p_d by (b) of Lemma A.2, and (g) follows from the definition of $\overline{d}_m$ in Lemma A.1. By Lemma A.1, $\overline{d}_m>h$ for all $m\in\{1,\dots,n\}$ when the Subsidy Condition is not satisfied. Therefore, $\pi^P\geq\pi^N$.

Compare customer demand. We first compare the demand associated with vendors building dedicated delivery fleets with that for the system without the platform. By (A.2), (7) and (9),

$$\lambda^D-\lambda^N=(p^D-h)-(p^N-h)=p^D-p^N\geq 0.$$

We then compare the demand associated with vendors participating on the platform with that for the system without the platform. By (A.4), (A.2), (6) and (9),

$$\lambda^P-\lambda^N=(p^P-p^d)-(p^N-h)=\frac{\left[\beta(n-m)+(2+\beta)[1-\beta(n-1)]\right](h-p_d^*)}{[2-\beta(n-1)](2+\beta)}.$$

By Proposition 1, $p_d^* < h$ is the Subsidy Condition is satisfied and $p_d^* \geq h$ otherwise. Therefore, $\lambda^P > \lambda^N$ if the Subsidy Condition is satisfied and $\lambda^P \leq \lambda^N$ otherwise.

Customer Surplus. If the Subsidy Condition is satisfied, all vendors participate on the platform in equilibrium by. As $p^P < p^N$ and $\lambda^P > \lambda^N$, by Definition 1, $u^P \succ u^D$. Otherwise, as $p^P \geq p^N$, $p^D \geq p^N$, $\lambda^P \leq \lambda^N$ and $\lambda^D \leq \lambda^N$, by Definition 1, $u^P \preceq u^D$.

B.3. Proof of Lemma 2

To show the robustness of our main results (Proposition 1 and Proposition 3) when considering the platform's profitability, we demonstrate scenarios in which the optimal value to the Platform Problem is positive, and $p_d^* < h$ and $p_d^* \geq h$ can arise. By doing so, we demonstrate the possibilities in which introducing the platform could either benefit vendors at the expense of customers, or conversely, benefit customers while hurting vendors, and the platform makes a positive profit. When $\frac{h}{\overline{w}} = \gamma \left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$, by Lemma A.1, $p_d^* = h$. Moreover, $m^* = n$ in equilibrium based on the refinement rule. By (A.11), we have

$$\begin{aligned}
\Pi(n, p_d^*) &= n\lambda(n, p_d^*)[\gamma(p^P(n, p_d^*) - p_d^*) + p_d^* - h] - \frac{h}{\overline{w}} \\
&= n\left(\frac{\theta - [1-\beta(n-1)]p_d^*}{2-\beta(n-1)}\right)\left[\gamma\left(\frac{\theta - [1-\beta(n-1)]p_d^*}{2-\beta(n-1)}\right) + p_d^* - h\right] - \frac{h}{\overline{w}} \\
&= n\gamma\left(\frac{\theta - [1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2 - \frac{h}{\overline{w}} \\
&= (n-1)\gamma\left(\frac{\theta - [1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2 \\
&> 0.
\end{aligned}$$

As $\Pi(n, \overline{d}_n)$ is continuous in $\overline{w}$, h and θ, the desired result follows.

C. Proofs for Various Extensions

In this section, we provide proofs for various extensions discussed in Section 5. Specifically, we investigate the underlying systems as described in Sections 5.1–5.5 in Sections C.1 – C.5 respectively.

C.1. Proof of Proposition 4

In this section, we analyze the system as described in Section 5.1 to highlight the role of substitutability among vendors. By (10) and letting $\theta' = \frac{\theta}{1+\delta}$, $\alpha' = \frac{1}{1-\delta^2}$ and $\beta' = \frac{\delta}{(n-1)(1-\delta^2)}$, we have $\lambda_i = \left(\theta' - \alpha' p_i + \beta' \sum_{j \neq i} p_j\right)^+$. We then note the following.

Firstly, by following the same analysis as those in Appendix A, we can obtain the following Lemma, which characterizes the equilibrium outcome when λ_i is defined as (10).

Lemma C.1. *Let*

$$g(\delta)=\frac{1-\delta}{(1+\delta)(2-\delta)^2}. \tag{C.1}$$

The equilibrium outcome is characterized as follows.

(i) If

$$\frac{h}{w}<\alpha'\gamma\left(\frac{\theta'-[\alpha'-\beta'(n-1)]h}{2\alpha'-\beta'(n-1)}\right)^2=\gamma(\theta-h)^2g(\delta), \tag{C.2}$$

there exists a unique equilibrium under which the platform subsidizes the delivery service per order, i.e., $p_d^*<h$, and all vendors participate on the platform in equilibrium, i.e., $m^*=n$.

(ii) Otherwise, the platform generates profit per order from the delivery service, i.e., $p_d^*\geq h$, and the equilibrium can be asymmetric with some vendors participating on the platform and the others building dedicated delivery fleets, i.e., $m^*\leq n$.

Secondly, by following the same analysis as that for the Proof of Proposition 3 in Appendix B.2, we can obtain the following results.

(i) If (C.2) is satisfied, we have $\pi^P<\pi^N$, $p^P<\pi^N$ and $\lambda^P>\lambda^N$, which implies $u^P\succ u^N$ by Definition 1.

(ii) Otherwise, we have $\pi^P\geq\pi^N$, $\pi^D\geq\pi^N$, $p^P\geq p^N$, $p^D\geq p^N$, $\lambda^P\leq\lambda^N$ and $\lambda^D\leq\lambda^N$, which implies $u^P\preceq u^N$ by Definition 1.

Lastly, we investigate conditions under which (C.2) is satisfied. By (C.1), $g'(\delta)=-\frac{2(2-\delta)(\delta^2-\delta+1)}{(1+\delta)^2(2-\delta)^4}<0$, and thus $g(\delta)$ is decreasing. Moreover, as $\lim_{\delta\to 1^-}\gamma(\theta-h)^2g(\delta)=0$, if $\gamma(\theta-h)^2g(0)=\frac{\gamma(\theta-h)^2}{4}>\frac{h}{w}$, there exists a threshold $\hat{\delta}\in(0,1)$ such that Condition (C.2) is satisfied if and only if $\delta\in[0,\hat{\delta})$. Otherwise, Condition (C.2) is not satisfied for all $\delta\in[0,1)$.

By combining the results derived above, we can obtain Proposition 4.

C.2. Proofs for Systems in Which Platform and Vendors Move Simultaneously

In this section, we analyze the system in which the platform and vendors move simultaneously and provide the proof of Proposition 5. We first conduct the equilibrium analysis. Define

$$G(p_d,m)=\frac{\theta+\frac{\beta(n-m)}{2-\beta(n-1)}\left(\theta+p_d+\frac{[2-\beta(m-1)](h-p_d)}{2+\beta}\right)}{2[1-\beta(m-1)]} - \frac{(1+\gamma)\left(\theta+\frac{\beta(n-m)(h-p_d)}{2+\beta}-[1-\beta(n-1)]p_d\right)}{2[2-\beta(n-1)]}+\frac{h}{2}-p_d, \tag{C.3}$$

and

$$H(m)=\frac{\beta^2m(n-m)}{(2+\beta)[1-\beta(m-1)]}+(1+\gamma)\left(\frac{\beta(n-1)}{2+\beta}+[1-\beta(n-1)]\right)-2[2-\beta(n-1)]. \tag{C.4}$$

Recall we define $\underline{d}_m$ for $m\in\{0,\dots,n-1\}$, and $\overline{d}_m$ for $m\in\{1,\dots,n\}$ in Lemma A.1. In Proposition C.1, we present the equilibrium result for the system in which the platform and vendor move simultaneously Proposition C.1.

Proposition C.1. *When the platform and vendors move simultaneously, the necessary and sufficient condition for the existence of an equilibrium with m vendors participating on the platform is as follows:*

$$G(\underline{d}_0, 0) \geq 0 \text{ for } m = 0,$$

$$G(\overline{d}_n, n) \leq 0 \text{ for } m = n, \quad \text{and}$$

$$\begin{cases} G(\underline{d}_m, m) \geq 0 \text{ and } G(\overline{d}_m, m) \leq 0 \text{ if } H(m) \leq 0, \\ G(\underline{d}_m, m) \leq 0 \text{ and } G(\overline{d}_m, m) \geq 0 \text{ if } H(m) \geq 0, \end{cases} \text{for } m \in \{1, \ldots, n-1\}.$$

Proof of Proposition C.1. Recall from the Proof of Lemma A.1 (step (1)), a strategy profile under which no vendor has an incentive to deviate given the platform's strategy p_d must be group-wise symmetric (vendors with the same platform participation decision play the same strategy). Therefore, it suffices to consider group-wise symmetric strategy profiles. Given m vendors participating on the platform and their food price p^F, and the (full) price p^D charged by vendors who build dedicated delivery fleets, the platform's profit can be rewritten as $m\lambda^P \left(\gamma p^F + p_d - h\right) - \frac{h}{w}$, where $\lambda^P = \left(\theta - p^P + \beta(n-m)p^D + \beta(m-1)p^P\right)^+$ by (1). Abusing notation, let

$$\Pi(p_d \mid m, p^F, p^D) = m\left[\theta - (1 - \beta(m-1))(p^F + p_d) + \beta(n-m)p^D\right]\left(\gamma p^F + p_d - h\right) - \frac{h}{w}.$$

Observe that $\Pi(p_d \mid p^F, p^D, m)$ is concave in p_d, with the maximum being achieved when p_d solves $LHS(p^F, p^D, m) = 0$, where $LHS(p^F, p^D, m) = \frac{\theta + \beta(n-k)p^D}{2[1-\beta(k-1)]} + \frac{h-(1+\gamma)p^F}{2} - p_d$. Recall we characterize the set of vendor equilibrium strategy profiles given a platform's strategy p_d in Lemma A.1. Then by (11), when the platform and vendors move simultaneously, the (necessary and sufficient) condition that ensures an equilibrium with m vendors participating on the platform is given by the existence of a p_d such that

$$LHS(p^F(m, p_d), p^D(m, p_d), m) = 0 \quad \text{and} \quad \begin{cases} p_d \geq \underline{d}_0, & \text{for } m = 0, \\ \underline{d}_m \leq p_d \leq \overline{d}_m, & \text{for } m \in \{1, \ldots, n-1\}, \\ p_d \leq \underline{d}_n, & \text{for } m = n, \end{cases} \tag{C.5}$$

where $p^F(m, p_d)$ and $p^D(m, p_d)$ are defined in (8) and (7) respectively. By some algebra, $LHS(p^F(m, p_d), p^D(m, p_d), m) = G(p_d, m)$, where $G(p_d, m)$ is define in (C.3). Observe that $G(p_d, m)$ is linear in p_d. Therefore, $G(p_d, m)$ is monotone. We can obtain that $\frac{\partial G(p_d, m)}{\partial p_d} = H(m)$, where $H(m)$ is define in (C.4). Because $H(0) = H(n) = (1+\gamma)\left(\frac{\beta(n-1)}{2+\beta} + [1 - \beta(n-1)]\right) - 2[2 - \beta(n-1)] < 0$, where the inequality follows from the dominant diagonal condition $\beta(n-1) < 1$, $G(p_d, m)$ is decreasing for $m \in \{0, n\}$. It follows that

- (C.5) is feasible if and only if $G(\underline{d}_0, 0) \geq 0$ for $m = 0$;

- (C.5) is feasible if and only if $G(\overline{d}_n, n) \leq 0$ for $m = n$; and
- for $m \in \{1, \ldots, n-1\}$, if $H(m) \leq 0$, $G(p_d, m)$ is decreasing in p_d and thus (C.5) is feasible if and only if $G(\underline{d}_m, m) \geq 0$ and $G(\overline{d}_m, m) \leq 0$; and if $H(m) \geq 0$, $G(p_d, m)$ is increasing in p_d and thus (C.5) is feasible if and only if $G(\underline{d}_m, m) \leq 0$ and $G(\overline{d}_m, m) \geq 0$.

□

We then prove Proposition 5 below based on the results obtained in Proposition C.1.

Proof of Proposition 5. By the Proposition 3, the introduction of the platform intensifies vendor competition and reduces vendor profits if and only if there exists an equilibrium with $p_d^* < h$. Assume (for contradiction) there exists an equilibrium with $p_d^* < h$ when the platform and vendors move simultaneously. By Lemma A.1, it suffices to consider the case where $m = n$ (i.e., $\mathcal{P}^*(m, p_d)$ with $m \in \{1, \ldots, n-1\}$ can be achieved only if $p_d > h$). Based on the Proof of Proposition C.1, the necessary condition of this is given by

$$p_d^* = \frac{\theta[1-\gamma(1-\beta(n-1))] + h[1-\beta(n-1)][2-\beta(n-1)]}{2[2-\beta(n-1)][1-\beta(n-1)] - (1+\gamma)[1-\beta(n-1)]^2} < h,$$

where p_d^* is the unique solution to

$$G(p_d, n) = \frac{\theta}{2[1-\beta(n-1)]} - \frac{(1+\gamma)\left[\theta - [1-\beta(n-1)]p_d\right]}{2[2-\beta(n-1)]} + \frac{h}{2} - p_d = 0.$$

By some algebra, this is equivalent to $\theta \leq [1-\beta(n-1)]h$, which violates the Vendor Viability Condition. □

C.3. Proof of Proposition 6

In this section, we analyze systems under the general contracts as described in Section 5.3 to identify the role played by the contract between the platform and vendors. The Proof of Proposition 6 consists of the following three steps.

Step (1). First, the problem solved by vendor i by participating on the platform can be transformed as

$$\max_{p_i} \quad \lambda_i(1-\gamma)\left[p_i - \left(p_d + \frac{p_t}{1-\gamma}\right)\right] + B.$$

As $p_i^F = p_i - p_d$, the platform's profit (13) can be rewritten as

$$\Pi(p_d, p_t) = \sum_{i=1}^{n} \lambda_i \left[\gamma p_i + (1-\gamma)\left(p_d + \frac{p_t}{1-\gamma}\right)\right] \cdot J_i - h \cdot \mu\left(w_p, \sum_{i=1}^{n} \lambda_i \cdot J_i\right) - B\sum_{i=1}^{n} \lambda_i \cdot J_i.$$

Therefore, we can change the decision variable by considering the combined term $x = p_d + \frac{p_t}{1-\gamma}$ instead of examining p_d and p_t separately.

Step (2). By following the Proof of Lemma 1, given $\mathcal{P}_{-i}$, vendor i chooses to participate on the platform if and only if $x \le X(M) = M - \left(\frac{(M-h)^2 - 4\left(\frac{h}{w}+B\right)}{1-\gamma}\right)^{\frac{1}{2}}$. Observe that $X(M)$ is identical to $d(M)$ defined in (3) by replacing $\frac{h}{w}$ with $\frac{h}{w}+B$.

Step (3). By following the same analysis as that for the Proof of Lemma A.1 and Proposition 1, we can obtain the following results.

(i) If (14) is satisfied, the platform's optimal strategy $x^* < h$ and all vendors participate on the platform in equilibrium, i.e., $m^* = n$.

(ii) Otherwise, the platform's optimal strategy $x^* \ge h$ and the equilibrium can be asymmetric with some vendors participating on the platform and the others building dedicated delivery fleets, i.e., $m^* \le n$.

Then, by the Proof of Proposition 3, the desired result follows.

C.4. Analyzing Systems with Asymmetric Vendor Scales

In this section, we examine systems with asymmetric vendor scales as described in Section 5.4. Define

$$d(M, s_i) = M - \left(\frac{(M-h)^2 - \frac{4h}{s_i \overline{w}}}{1-\gamma}\right)^{\frac{1}{2}}. \tag{C.6}$$

By following the Proof of Lemma 1, given $\mathcal{P}_{-i}$, the set of vendor i's best response strategies is given by

$$\mathcal{B}_i^*(\mathcal{P}_{-i}) = \begin{cases} \{(1, p_i^P)\} & \text{if } p_d < d(M(\mathcal{P}_{-i}), s_i), \\ \{(0, p_i^D)\} & \text{if } p_d > d(M(\mathcal{P}_{-i}), s_i), \\ \{(1, p_i^P), (0, p_i^D)\} & \text{if } p_d = d(M(\mathcal{P}_{-i}), s_i), \end{cases} \tag{C.7}$$

where p_i^P and p_i^D are define in (4), and the function $M(\mathcal{P}_{-i})$ is defined in (2). Recall we define the set of second-stage equilibrium profiles in (5). For convenience, we use $\mathcal{S}$ to denote the set of vendors participating on the platform, and m denote the number of vendors participating on the platform (i.e., the number of elements in $\mathcal{S}$). Moreover, by abusing a bit notation, we use

$$P^*(\mathcal{S}, p_d) = (p^P(m, p_d), p^D(m, p_d), \mathcal{S})$$

to denote the vendor strategy profile under which the set of vendors participating on the platform is given by $\mathcal{S}$, the (full) price associated with vendors participating on the platform is given by $p^P(m, p_d)$ and that associated with vendors building dedicated delivery fleets is $p^D(m, p_d)$, where $p^P(m, p_d)$ and $p^D(m, p_d)$ are defined in (6) and (7) respectively. We then introduce some preliminary results per Lemma C.2 below.

Lemma C.2. *For $m \in \{0, \ldots, n-1\}$, $d(\underline{M}_m(p_d), s_i) = p_d$ admits a unique solution $\underline{d}_{m,i}$ with respect to p_d; and for $m \in \{1, \ldots, n\}$, $d(\overline{M}_m(p_d), s_i) = p_d$ admits a unique solution $\overline{d}_{m,i}$ with respect to p_d, then* *(i)* *if* $\frac{h}{s_1\overline{w}} < \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$ *is satisfied, we have* $\underline{d}_{0,i} < \cdots < \overline{d}_{m-1,i} < \underline{d}_{m-1,i} < \overline{d}_{m,i} < \underline{d}_{m,i} < \cdots < \overline{d}_{n,i} < h$ *for* $i \in \{1,\ldots,n\}$, $\underline{d}_{m,n} \leq \cdots \leq \underline{d}_{m,2} \leq \underline{d}_{m,1} < h$ *for* $m \in \{0,\ldots,n-1\}$, *and* $\overline{d}_{m,n} \leq \cdots \leq \overline{d}_{m,2} \leq \overline{d}_{m,1} < h$ *for* $m \in \{1,\ldots,n\}$*; and*

(ii) *if* $\frac{h}{s_n\overline{w}} \geq \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$ *is satisfied, we have* $h \leq \overline{d}_{n,i} \leq \cdots \leq \underline{d}_{m,i} \leq \overline{d}_{m,i} \leq \underline{d}_{m-1,i} \leq \overline{d}_{m-1,i} \cdots \leq \underline{d}_{0,i}$ *for* $i \in \{1,\ldots,n\}$, $h \leq \underline{d}_{m,n} \leq \cdots \leq \underline{d}_{m,2} \leq \underline{d}_{m,1}$ *for* $m \in \{0,\ldots,n-1\}$ *and* $h \leq \overline{d}_{m,n} \leq \cdots \leq \overline{d}_{m,2} \leq \overline{d}_{m,1}$ *and* $m \in \{1,\ldots,n\}$.

Then we have the following results on the set of second-stage equilibrium profiles $\Omega^*(p_d)$ *with asymmetric vendor scales:*

- *For* $\mathcal{S} = \emptyset$, $\mathcal{P}^*(\mathcal{S}, p_d) \in \Omega^*(p_d)$ *if and only if* $p_d \geq \overline{d}_{n,1}$;
- *For* $\mathcal{S} = \{1,\ldots,n\}$, $\mathcal{P}^*(\mathcal{S}, p_d) \in \Omega^*(p_d)$ *if and only if* $p_d \leq \overline{d}_{n,n}$*; and*
- *For* $\mathcal{S} \subseteq \{1,\ldots,n-1\}$, $\mathcal{P}^*(\mathcal{S}, p_d) \in \Omega^*(p_d)$, *if and only if* $\underline{d}_{m,j} \leq p_d \leq \overline{d}_{m,i}$ *for all* $i \in \mathcal{S}$ *and* $j \notin \mathcal{S}$.

Proof of Lemma C.2. The proof consists of the following two steps.

Step (1). We prove the first part of results in Lemma C.2 on $\underline{d}_{m,i}$ and $\overline{d}_{m,i}$. We consider the following cases.

Case (i) $\frac{h}{s_1\overline{w}} < \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$. This condition is equivalent to $d(\overline{M}_n(h), s_1) < h$. Then we can obtain that: (1) $\underline{d}_{0,1} < \cdots < \overline{d}_{m-1,1} < \underline{d}_{m-1,1} < \overline{d}_{m,1} < \underline{d}_{m,1} < \cdots < \overline{d}_{n,1} < h$ by Lemma A.1; and (2) $\overline{d}_{n,n} \leq \cdots \leq \overline{d}_{n,2} \leq \overline{d}_{n,1} < h$ as $d(\overline{M}_n(p_d), s_i)$ is decreasing in s_i. Then (1) and (2) together with Lemma A.1 implies the case (i) results of Lemma C.2.

Case (ii) $\frac{h}{s_n\overline{w}} \geq \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$. This condition is equivalent to $d(\overline{M}_n(h), s_n) \geq h$. Then we can obtain that: (1) $h \leq \overline{d}_{n,n} \leq \cdots \leq \underline{d}_{m,n} \leq \overline{d}_{m,n} \leq \underline{d}_{m-1,n} \leq \overline{d}_{m-1,n} \leq \cdots \leq \underline{d}_{0,n}$ by Lemma A.1; and (2) $h \leq \overline{d}_{n,n} \leq \cdots \leq \overline{d}_{n,2} \leq \overline{d}_{n,1}$. Then (1) and (2) together with Lemma A.1 implies the case (ii) results of Lemma C.2.

Step (2). We prove the second part of results in Lemma C.2 on $\Omega^*(p_d)$. By following Step (1) and Step (2) for the Proof of Lemma A.1, given that there are m vendors participating on the platform (irrespective of their scales), the conditions that ensure no within-group deviations are as follows: the price associated with vendors participating on the platform is given by $p^P(m, p_d)$ and that associated with vendors building dedicated delivery fleets is given by $p^D(m, p_d)$, where $p^P(m, p_d)$ and $p^D(m, p_d)$ satisfy (6) and (7) respectively. Moreover, the condition that ensures no cross-group deviations is given by

$$\begin{cases} d(\underline{M}_m(p_d), s_j) \leq p_d \leq d(\overline{M}_m(p_d), s_i) \text{ for } i \in \mathcal{S} \text{ and } j \notin \mathcal{S} & \text{if } m \in \{1,\ldots,n-1\}, \\ p_d \geq d(\underline{M}_0(p_d), s_1) & \text{if } m = 0, \\ p_d \leq d(\overline{M}_n(p_d), s_n) & \text{if } m = n, \end{cases} \tag{C.8}$$

where $\underline{M}_m(p_d)$ and $\overline{M}_m(p_d)$ are defined in (A.6) and (A.7) respectively. The reason is as follows. For $m \in \{1,\ldots,n-1\}$, we need to ensure that vendor i has no incentive to exit the platform for all $i \in \mathcal{S}$, and vendor j has no incentive to participate on the platform for all $j \notin \mathcal{S}$. Given there are m vendors participating on the platform, $M(\mathcal{P}^{-j}) = \underline{M}_m(p_d)$ and $M(\mathcal{P}_{-i}) = \overline{M}_m(p_d)$, then by (C.7), this condition is given by $d(\underline{M}_m(p_d), s_j) \leq p_d \leq d(\overline{M}_m(p_d), s_i)$ for all $i \in \mathcal{S}$ and $j \notin \mathcal{S}$. For $m = 0$, we need to ensure that no vendor has an incentive to participate on the platform. By (C.7), the condition of which is given by $p_d \geq d(\underline{M}_0(p_d), s_i)$ for all $i \in \{1,\ldots,n\}$. As $d(M, s_i)$ is decreasing in s_i, this condition reduces to $p_d \geq d(\underline{M}_0(p_d), s_1)$. For $m = n$, we need to ensure that no vendor has an incentive to exit the platform. By (C.7), the condition of which is given by $p_d < d(\overline{M}_n(p_d), s_i)$ for all $i \in \{1,\ldots,n\}$, which reduces to $p_d < d(\overline{M}_n(p_d), s_n)$. Therefore, by the definitions of $\underline{d}_{m,i}$ and $\overline{d}_{m,i}$, we can obtain the results on $\Omega^*(p_d)$ in Lemma C.2. □

We close this section by providing the Proof of Proposition 7.

Proof of Proposition 7. The analysis consists of the following two steps.

Step (1). We characterize the the platform's optimal strategy. The platform's profit in the presence of asymmetric vendor scales is given by

$$\Pi(\mathcal{S}, p_d) = \Pi(m, p_d) \cdot \sum_{i \in \mathcal{S}} s_i,$$

where $\Pi(m, p_d)$ is defined in (A.11). Following the Proof of Proposition 1, the problem solved by the platform reduces to

$$p_d^* = \underset{\mathcal{S} \subseteq \{1,\ldots,n\}}{\arg\max}\ \Pi(\mathcal{S}, p_d^*(\mathcal{S})),$$

where

$$p_d^*(\mathcal{S}) = \begin{cases} \tilde{p}_d(m), & \text{if } \tilde{p}_d(m) \in (\max_{j \notin \mathcal{S}} \underline{d}_{m,j}, \min_{i \in \mathcal{S}} \overline{d}_{m,i}), \\ \max_{j \notin \mathcal{S}} \underline{d}_{m,j}, & \text{if } \tilde{p}_d(m) \leq \max_{j \notin \mathcal{S}} \underline{d}_{m,j}, \\ \min_{i \in \mathcal{S}} \overline{d}_{m,i}, & \text{if } \tilde{p}_d(m) \geq \min_{i \in \mathcal{S}} \overline{d}_{m,i}, \end{cases} \quad \text{for } \mathcal{S} \subseteq \{1,\ldots,n-1\}, \text{ and} \tag{C.9}$$

$$p_d^*(\mathcal{S}) = \begin{cases} \tilde{p}_d(n), & \text{if } \tilde{p}_d(n) < \overline{d}_{n,n}, \\ \overline{d}_{n,n}, & \text{otherwise}, \end{cases} \quad \text{for } \mathcal{S} = \{1,\ldots,n\}, \tag{C.10}$$

and $\tilde{p}_d(m)$ is the unique solution to $\frac{\partial \Pi(m,p_d)}{\partial p_d} = 0$. We then consider the following two cases.

Case (i) $\frac{h}{s_1 \overline{w}} < \gamma \left(\frac{\theta - [1-\beta(n-1)]h}{2-\beta(n-1)} \right)^2$. In this case, we have $p_d^* < h$ in equilibrium. This result follows from (C.9) – (C.10) and the fact that $\underline{d}_{m,i} < h$ for all $m \in \{0,\ldots,n-1\}$ and $i \in \{1,\ldots,n\}$ and $\overline{d}_{m,i} < h$ for all $m, i \in \{1,\ldots,n\}$ by Lemma C.2.

Case (ii) $\frac{h}{s_n\overline{w}} \geq \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$. In this case, we have $p_d^* \geq h$ in equilibrium. This result follows from (C.9) – (C.10) and the fact that $\underline{d}_{m,i} \geq h$ for all $m \in \{0,\ldots,n-1\}$ and $i \in \{1,\ldots,n\}$ and $\overline{d}_{m,i} \geq h$ for all $m,i \in \{1,\ldots,n\}$ by Lemma C.2.

Step (2). We compare the equilibrium outcomes for systems with and without the platform. We use p_i^P, p_i^D, and p_i^N to denote the full price associated with vendor i by participating on the platform, building a dedicated delivery fleet, and in the system without the platform, respectively. Let π_i^P, π_i^D and π_i^N, and λ_i^P, λ_i^D and λ_i^N be similarly defined. In an equilibrium with m vendors participating on the platform, we have $p_i^P = p^P(m,p_d^*)$, $p_i^D = p^D(m,p_d^*)$, $p_i^N = p^N$, $\lambda_i^P = s_i\lambda^P(m,p_d^*)$, $\lambda_i^D = s_i\lambda^D(m,p_d^*)$ and $\lambda_i^N = s_i\lambda^N$, where p_d^* is the platform's equilibrium strategy. Then following the Proof of Proposition 3, $p_i^P < p_i^N$, $p_i^D < p_i^N$, $\pi_i^D < \pi_i^N$, $\lambda_i^P > \lambda^N$ and $\lambda_i^D < \lambda_i^N$ when $\frac{h}{s_1\overline{w}} < \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$ as $p_d^* < h$ from step (1), and $p_i^P \geq p_i^N$, $p_i^D \geq p_i^N$, $\pi_i^D \geq \pi_i^N$, $\lambda_i^P \leq \lambda^N$ and $\lambda_i^D \leq \lambda_i^N$ when $\frac{h}{s_n\overline{w}} \geq \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$ as $p_d^* \geq h$ from step (1). Then, it remains to compare π_i^P and π_i^N. Following the proof of Proposition 3, in an equilibrium with m vendors participating on the platform,

$$\frac{\pi_i^P - \pi_i^N}{s_i} = (1-\gamma)\left(\frac{\overline{M}_m(p_d^*) - p_d^*}{2}\right)^2 - \left(\lambda^D(p^D - h) - \frac{h}{s_i\overline{w}}\right).$$

We then consider the following two cases.

Case (i) $\frac{h}{s_1\overline{w}} < \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$. Firstly, we notice that a second-stage equilibrium profile with m vendors participating on the platform must adhere to the following pattern: Vendors $\{1,\ldots,m\}$ participate on the platform with a (full) price $p^P(m,p_d)$, and vendors $\{m+1,\ldots,n\}$ establish dedicated delivery fleets with a (full) price $p^D(m,p_d)$. This is because $\underline{d}_{m,j} \leq \overline{d}_{m,i}$ only if $j > i$ by Lemma C.2. By (C.9), there does not exist a second-stage equilibrium profile under which vendor i builds a dedicated delivery fleet and vendor j participates on the platform for $i < j$.

Secondly, we show that in an equilibrium with m vendors participating on the platform, we must have $p_d^* = \overline{d}_{m,m}$. The reason is as follows. We have $\tilde{p}_d(m) > h$ from (A.14) and $\underline{d}_{m,i} < h$ for all $m \in \{1,\ldots,n-1\}$ and $i \in \{1,\ldots,n\}$ from Lemma C.2. By (C.9) – (C.10), $p_d^* = \min_{i\in\mathcal{S}} \overline{d}_{m,i} = \min_{i\in\{1,\ldots m\}} \overline{d}_{m,i} = \overline{d}_{m,m}$, where the last equality follows from Lemma C.2.

Lastly, we conduct the comparison. For vendor m, we have

$$\begin{aligned}\frac{\pi_i^P - \pi_i^N}{s_m} &= (1-\gamma)\left(\frac{\theta + \overline{M}_m(\overline{d}_{m,m}) - \overline{d}_{m,m}}{2}\right)^2 - \frac{1}{s_m}\left(\lambda^D(p^D - h) - \frac{h}{\overline{w}}\right)\\ &\overset{(a)}{=} \left(\frac{\theta + \overline{M}_m(\overline{d}_{m,m}) - h}{2}\right)^2 - \frac{h}{s_m\overline{w}} - \frac{1}{s_m}\left(\lambda^D(p^D - h) - \frac{h}{\overline{w}}\right)\end{aligned}$$

$$\stackrel{(b)}{=} \frac{1}{4}\left(\frac{2\theta + \frac{\beta[2(m-1)+\beta(n-1)]\overline{d}_{m,m}+2\beta(n-m)h}{2+\beta}}{2-\beta(n-1)} - h\right)^2 - \left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$$
$$\stackrel{(c)}{<} 0,$$

where (a) follows from the definition of $\overline{d}_{m,m}$ in Lemma C.2, (b) follows from the definition of $\overline{M}_m(p_d)$ in (A.6), and (c) follows from $\overline{d}_{m,m} < h$ by Lemma C.2. For vendor i, where $i \in \{1,\ldots,m-1\}$, $\frac{\pi_i^P-\pi_i^N}{s_i} = (1-\gamma)\left(\frac{\theta+\overline{M}_m(\overline{d}_{m,m})-\overline{d}_{m,m}}{2}\right)^2 - \frac{1}{s_i}\left(\lambda^D(p^D-h)-\frac{h}{\overline{w}}\right)$ can be either positive or negative. As $s_i \le s_j$ if $i<j$, $\pi_j^P - \pi_j^N \ge 0$ implies that $\pi_i^P - \pi_i^N \ge 0$. Therefore, either $\pi_i^P < \pi_i^N$ for all $i \in \{1,\ldots,m\}$, or there exists a $k \in \{1,\ldots,m-1\}$ such that $\pi_i^P \ge \pi_i^N$ for $i \in \{1,\ldots,k\}$ and $\pi_i^P < \pi_i^N$ for $i \in \{k+1,\ldots,m\}$.

Case (ii) $\frac{h}{s_n\overline{w}} \ge \gamma\left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$. In this case, we have

$$\frac{\pi_i^P-\pi_i^N}{s_i} \stackrel{(d)}{\ge} (1-\gamma)\left(\frac{\overline{M}_m(\overline{d}_{m,i})-\overline{d}_{m,i}}{2}\right)^2 - \frac{1}{s_i}\left(\lambda^D(p^D-h)-\frac{h}{\overline{w}}\right)$$
$$= \left(\frac{\overline{M}_m(\overline{d}_{m,i})-h}{2}\right)^2 - \frac{h}{s_i\overline{w}} - \frac{1}{s_i}\left(\lambda^D(p^D-h)-\frac{h}{\overline{w}}\right)$$
$$= \frac{1}{4}\left(\frac{2\theta + \frac{\beta[2(m-1)+\beta(n-1)]\overline{d}_{m,i}+2\beta(n-m)h}{2+\beta}}{2-\beta(n-1)} - h\right)^2 - \left(\frac{\theta-[1-\beta(n-1)]h}{2-\beta(n-1)}\right)^2$$
$$\stackrel{(e)}{\ge} 0,$$

where (d) follows from $p_d^* \le \min_{i\in\mathcal{S}} \overline{d}_{m,i}$ by (C.9)–(C.10) and $\overline{M}_m(p_d) - p_d$ being decreasing in p_d for all $m \in \{1,\ldots,n\}$ by (b) of Lemma A.2, and (e) follows from $\overline{d}_{m,i} \ge h$ by Lemma C.2. □

C.5. Proof of Proposition 8

In this section, we analyze systems in which couriers receive a piecemeal take-home-pay per order as described in Section 5.5. The Proof of Proposition 8 consists of the following four steps.

Step (1). We solve for the best response strategy of vendor i. Let $\hat{p}_i^P = \frac{M(\mathcal{P}_{-i})+p_d}{2}$ and $\hat{p}_i^D = \frac{M(\mathcal{P}_{-i})+\hat{h}}{2}$, and define

$$\hat{d}(M) = M - \frac{M-\hat{h}}{\sqrt{1-\gamma}}.$$

By following the same analysis as that of Lemma 1, given the delivery fee charged by the platform p_d and the strategy profile of all other vendors $\mathcal{P}_{-i}$, the set of best response strategies of vendor i is given by

$$\mathcal{B}_i^*(\mathcal{P}_{-i}) = \begin{cases} \{(1,\hat{p}_i^P)\} & \text{if } p_d < \hat{d}(M(\mathcal{P}_{-i})), \\ \{(0,\hat{p}_i^D)\} & \text{if } p_d > \hat{d}(M(\mathcal{P}_{-i})), \\ \{(1,\hat{p}_i^P),(0,\hat{p}_i^D)\} & \text{if } p_d = \hat{d}(M(\mathcal{P}_{-i})). \end{cases}$$

Observe that vendor i chooses to participate on the platform only if $p_d < \hat{h}$.

Step (2). We characterize the set of second-stage equilibrium strategies $\hat{\Omega}^*(p_d)$, given the delivery fee p_d charged by the platform. By following the same analysis as that for no within-group deviation in the Proof of Lemma A.1, provided that m vendors participating on the platform, the (full) prices associated vendors participating on the platform and building dedicated delivery fleets are respectively given by

$$\hat{p}^P(m,p_d) = \frac{\theta + p_d + \frac{\beta(n-m)(\hat{h}-p_d)}{2+\beta}}{2-\beta(n-1)} \quad \text{and} \quad \hat{p}^D(m,p_d) = \frac{\theta + p_d + \frac{[2-\beta(m-1)](\hat{h}-p_d)}{2+\beta}}{2-\beta(n-1)}.$$

Then by following the same analysis as that for no cross-group deviation in the Proof of Lemma A.1, provided that m vendors participating on the platform, the condition that ensures no cross-group deviations is given by

$$\begin{cases} p_d \geq \hat{d}(\underline{\hat{M}}_0(p_d)) & \text{if } m=0, \\ \hat{d}(\underline{\hat{M}}_m(p_d)) \leq p_d \leq \hat{d}(\hat{\overline{M}}_m(p_d)) & \text{if } m \in \{1,\dots,n-1\}, \\ p_d \leq \hat{d}(\hat{\overline{M}}_n(p_d)) & \text{if } m=n, \end{cases} \tag{C.11}$$

where $\underline{\hat{M}}_m(p_d) = \frac{2\theta + \beta\left(\frac{2mp_d + [2(n-m-1)+\beta(n-1)]\hat{h}}{2+\beta}\right)}{2-\beta(n-1)}$ for $m \in \{0,\dots,n-1\}$ and $\hat{\overline{M}}_m(p_d) = \frac{2\theta + \beta\left(\frac{[2(m-1)+\beta(n-1)]p_d + 2(n-m)\hat{h}}{2+\beta}\right)}{2-\beta(n-1)}$ for $m \in \{1,\dots,n\}$.

Step (3). We solve for the optimal strategy for the platform and characterize the equilibrium outcomes. From Step (1), it suffices to consider $p_d < \hat{h}$. Observe that $\hat{d}(M)$ is decreasing in M and $\underline{\hat{M}}_m(p_d)$ and $\hat{\overline{M}}_m(p_d)$ are increasing in p_d, it follows that $\hat{d}(\underline{\hat{M}}_m(p_d)) - p_d$ and $\hat{d}(\hat{\overline{M}}_m(p_d)) - p_d$ are decreasing in p_d. Given $p_d < \hat{h}$, for $m \in \{1,\dots,n-1\}$, $\hat{d}(\underline{\hat{M}}_m(p_d)) > \hat{d}(\hat{\overline{M}}_m(p_d))$, and thus Condition (C.11) is not satisfied. It follows that there does not exists a second-stage equilibrium with $m \in \{1,\dots,n-1\}$ vendors participating on the platform. Therefore, it suffices to focus on the case where $m = n$. Then the problem solved by the platform can be reduced to:

$$\begin{aligned} \max_{p_d} \quad & \hat{\Pi}(p_d) = n\hat{\lambda}^P(n,p_d)\left[\gamma \hat{p}^P(n,p_d) + (1-\gamma)p_d - \hat{h}\right] \\ \text{subject to} \quad & p_d \leq \hat{d}(\hat{\overline{M}}_n(p_d)), \end{aligned}$$

where $\hat{\lambda}^P(n,p_d)$ denote the demand rate when the full prices associate with each vendor is given by $\hat{p}^P(n,p_d)$. We then show that the optimal solution p_d^* to the above problem is given by the unique solution to

$$\hat{d}(\hat{\overline{M}}_n(p_d)) - p_d = \frac{2\theta + \beta(n-1)p_d}{2-\beta(n-1)} - \frac{\frac{2\theta+\beta(n-1)p_d}{2-\beta(n-1)} - \hat{h}}{\sqrt{1-\gamma}} - p_d = 0.$$

We begin by showing that $\hat{d}(\hat{\overline{M}}_n(p_d)) - p_d = 0$ admits a unique solution in $[-\frac{2\theta}{\beta(n-1)}, \hat{h}]$. As $\hat{d}(\overline{M}_n(h)) - h < 0$, $\hat{d}(\overline{M}_n(-\frac{2\theta}{\beta(n-1)})) + \frac{2-\theta}{\beta(n-1)} = \frac{\hat{h}}{\sqrt{1-\gamma}} + \frac{2\theta}{\beta(n-1)} > 0$, and $\hat{d}(\overline{M}_n(p_d)) - p_d$ is decreasing, by the Intermediate value theorem, the desired result follows. Then by following the same analysis as that for the derivation of (A.14) in the Proof of Proposition 1, we can show that the platform's profit function $\hat{\Pi}(p_d)$ is concave in p_d. Moreover, provided that vendors choose to operate in the delivery channel in the absence of the platform, the unique solution $\tilde{p}_d$ to the first order condition $\hat{\Pi}'(p_d) = 0$ satisfies $\tilde{p}_d > \hat{h}$. Therefore, the optimal solution to the above problem is given by the highest value of p_d such that the constrain $p_d \le \hat{d}(\hat{\overline{M}}_n(p_d))$ holds, which is given by the unique solution to $\hat{d}(\hat{\overline{M}}_n(p_d)) = p_d$ as $\hat{d}(\hat{\overline{M}}_n(p_d)) - p_d$ is decreasing in p_d. Then in equilibrium, the (full) price associated with each vendor is given by $p^P = \hat{p}^P(n, p_d)$.

Step (4). We compare prices, demand rates, and vendor profits for systems with and without the platform. In the system without the platform, by following the same analysis as that in the Proof of Proposition 2, the full price associated with each vendor in the system without the platform is given by $p^N = \frac{\theta+\hat{h}}{2-\beta(n-1)}$. As all vendors participate on the platform in equilibrium by the analysis from previous steps, it suffices to compare p^N (λ^N and π^N) with p^P (λ^P and π^P).

Compare (full) price. We can obtain that $p^P - p^N = \frac{\theta+p_d}{2-\beta(n-1)} - \frac{\theta+\hat{h}}{2-\beta(n-1)} = \frac{p_d^*-h}{2-\beta(n-1)} < 0$ as $p_d^* < \hat{h}$.

Compare customer demand. The first order condition of Problems (P) and (15) imply that $\lambda^P = p^P - p_d^*$ and $\lambda^N = p^N - \hat{h}$. Therefore, we have $\lambda^P - \lambda^N = \frac{\theta-[1-\beta(n-1)]p_d^*}{2-\beta(n-1)} - \frac{\theta-[1-\beta(n-1)]\hat{h}}{2+\beta} = \frac{[1-\beta(n-1)](\hat{h}-p_d^*)}{2+\beta} > 0$ as $p_d^* < \hat{h}$. By Definition 1, $u^P \succ u^N$.

Compare vendor profit. We have

$$
\begin{aligned}
\pi^P - \pi^N &= (1-\gamma)\lambda^P(p^P - p_d^*) - \lambda^N(p^N - \hat{h}) \\
&= (1-\gamma)(p^P - p_d^*)^2 - (p^N - \hat{h})^2 \\
&= (1-\gamma)\left(\frac{\overline{M}_n(p_d^*) - p_d^*}{2}\right)^2 - \left(\frac{\theta - [1-\beta(n-1)]\hat{h}}{2-\beta(n-1)}\right)^2 \\
&\overset{(a)}{=} \left(\frac{\overline{M}_n(p_d^*) - \hat{h}}{2}\right)^2 - \left(\frac{\theta - [1-\beta(n-1)]\hat{h}}{2-\beta(n-1)}\right)^2 \\
&= \frac{1}{4}\left(\frac{2\theta - [2-\beta(n-1)]\hat{h} + \beta(n-1)p_d^*}{2-\beta(n-1)}\right)^2 - \left(\frac{\theta - [1-\beta(n-1)]\hat{h}}{2-\beta(n-1)}\right)^2 \\
&< 0,
\end{aligned}
$$

where (a) follows from the fact that p_d^* is the unique solution to $\hat{d}(\hat{\overline{M}}_n(p_d)) - p_d = 0$ and the last inequality follows from $p_d^* < \hat{h}$.